Global Networks, Monetary Policy and Trade*

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Abstract

We develop a novel framework to study the interaction between monetary policy and trade. Our New Keynesian open economy model incorporates international production networks, sectoral heterogeneity in price rigidities, and trade distortions. We decompose the general equilibrium response to trade shocks into distinct channels that account for demand shifts, policy effects, exchange rate adjustments, expectations, price stickiness, and input–output linkages. Tariffs act simultaneously as demand and supply shocks, leading to endogenous fragmentation through changes in trade and production network linkages. We show that the net impact of tariffs on domestic inflation, output, employment, and the dollar depends on the endogenous monetary policy response in both the tariff-imposing and tariff-exposed countries, within a global general equilibrium framework. Our quantitative exercise replicates the observed effects of the 2018 tariffs on the U.S. economy and predicts a *1.6* pp decline in U.S. output, a *0.8* pp rise in inflation, and a *4.8*% appreciation of the dollar in response to a retaliatory trade war linked to tariffs announced on "Liberation Day." Tariff threats, even in the absence of actual implementation, are self-defeating—leading to a *4.1*% appreciation of the dollar, *0.6*% deflation, and a *0.7* pp decline in output, as agents re-optimize in anticipation of future distortions. Dollar appreciates less or even can depreciate under retaliation, tariff threats, and increased global uncertainty.

JEL Codes: E2, E3, E6, F1, F4
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1 Introduction

The impact of tariffs on inflation and economic activity has been widely studied, yet there remains active debate regarding their net effects—particularly in the context of the U.S. tariff policies announced in 2025. The prevailing New Keynesian (NK) literature typically models tariffs as short-run demand shocks, often overlooking their implications for producer prices and production costs. In contrast, standard trade models primarily focus on long-run welfare outcomes. This approach omits both the short-run demand channel and the dynamic adjustments driven by forward-looking behavior of prices and exchange rates in general equilibrium. Our work extends the standard NK framework by incorporating both demand- and supply-side channels, offering a more comprehensive view of how tariffs propagate through an economy with full input–output (I–O) linkages to global production networks.

Our New Keynesian open economy (NKOE) model features country–sector heterogeneity in price rigidities and monetary policy preferences within global trade and production networks. We ask three central questions: (i) What are the key channels through which tariffs affect inflation, output, and other macroeconomic aggregates such as trade deficits and employment? (ii) Can tariffs be both inflationary and contractionary in the short run? What roles do monetary policy, exchange rate adjustment, and inflation expectations play in shaping these outcomes? (iii) How do the answers to these questions depend on international production networks—a feature typically absent from standard NKOE frameworks? To answer these questions, we provide both analytical solutions and quantitative estimates.

We show that tariffs operate as both demand and supply shocks. To understand how these effects are transmitted through the economy, we decompose their net impact into three broad and seven granular channels. While some of these mechanisms are well-documented in the existing literature, others represent novel contributions, offering new theoretical insights and quantitatively meaningful results. The three broad channels are: (i) demand reallocation between domestic and foreign goods; (ii) endogenous monetary policy responses—potentially asymmetric—arising from our global general equilibrium framework as opposed to standard two-country models; and (iii) exchange rate adjustment, which may also be asymmetric across trading partners. To the best of our knowledge, this paper is the first to examine these channels jointly within a global general equilibrium setting.

We can decompose these three broad channels into the following seven granular channels, both theoretically and quantitatively: (i) There is a direct impact on consumer prices—a well-known and widely documented channel in the data (e.g., Amiti et al., 2019; Fajgelbaum et al., 2020; Cavallo et al., 2021; Cox, 2021; Fajgelbaum and Khandelwal, 2022). (ii) There is also a direct impact on producers' marginal costs—a much less studied channel (see, for example, Caliendo and Parro, 2015). Tariffs may be imposed on any sector—such as steel and aluminum, automobiles, or even services—where producers' marginal costs are interconnected through input–output (I–O) linkages. The quantitative impact of both (i) and (ii) depends on the elasticity of substitution between home and foreign goods, as well as between domestic and imported inputs. (iii) Tariffs indirectly reduce contemporaneous aggregate demand, as higher prices lower demand in the presence of an inflation-targeting central bank. This channel has also been emphasized by Bianchi and Coulibaly (2025), Monacelli (2025), and Bergin and Corsetti (2023). (iv) International capital flows allow the exchange rate to appreciate in the tariff-imposing country as demand shifts from foreign to domestic goods, partially offsetting the inflationary pressure. However, the extent of this offset—and the exchange rate adjustment more broadly—depends on the endogenous responses of monetary policy in both the tariff-imposing and tariff-exposed countries. This mechanism constitutes a novel channel we highlight. (v) Forward-looking agents endogenously adjust their inflation expectations in response to tariffs, which increases current inflation. This is a standard New Keynesian mechanism that we extend to the context of tariff shocks. (vi) Country–sector heterogeneity in price stickiness generates downward pressure on aggregate prices due to the unequal transmission of shocks across sectors. (vii) Propagation through the production network depends on input–output linkages. These last three channels are derived from our analytical solution for NKOE-Leontief inverse. This dynamic general equilibrium object, new to our paper, summarizes the net effects of country–sector heterogeneity in price stickiness and monetary policy preferences, including varying weights on inflation and output in Taylor rules.

Whether tariffs are inflationary and contractionary in the short run depends on the relative magnitudes and signs of the underlying transmission channels. Intuitively, if the supply shock component dominates the demand-side effects, tariff shocks will simultaneously raise inflation and reduce output. Conversely, if the demand-side component, inclusive of the monetary policy response, prevails, tariffs will be deflationary and contractionary. We find that the deflationary outcome is more likely under "tariff threats", where demand contracts before the direct inflationary pressure from actual tariffs take effect in the future. Although several channels, such as, lower contemporaneous demand, the exchange rate offset, and price stickiness puts downward pressure on inflation together with a central bank putting higher weight on inflation in Taylor rule restraining aggregate demand, we still find tariffs to be inflationary, in general, which highlights the importance of both direct effects and the supply shock nature of tariffs.

How do global networks mute or amplify inflation? We demonstrate that the network channel can amplify the influence of more rigid or more flexible sectors, thereby exerting downward or upward pressure on inflation. Intuitively, this result stems from sectoral interlinkages and underscores the importance of modeling the full global input–output (I–O) network, rather than relying solely on a simple tradable vs. non-tradable sector distinction. This finding extends the closed-economy result from Rubbo (2023), who shows that when many sectors are interlinked, the aggregate Phillips Curve flattens—muting price responses while amplifying quantity adjustments. While we observe a similar effect, our result is grounded in the structure of global production networks and the heterogeneous price stickiness parameters we use. We provide an analytical explanation for how interlinked global supply chains can dampen inflationary responses in a given country. At the same time, we also identify the opposite propagation channel: input–output linkages can, under certain conditions, exert upward pressure on inflation. Although increasing the number of sectors and having a more granular network can flatten the Phillips Curve, the very fact that firms' marginal costs depend on producer prices in other sectors can steepen the Phillips Curve for a given network. This dependency transmits cost pressures across sectors, amplifying price responses and contributing to inflation.

Our key contribution to the closed-economy network literature—which is predominantly static or two-period in nature—is to formulate a dynamic open-economy setting and derive analytical solutions for NKOE Leontief inverse that accommodates heterogeneity in sectoral price stickiness as well as cross-country differences in monetary policy preferences and inflation expectations in shaping the transmission of trade shocks. The NKOE Leontief inverse, $\tilde{\boldsymbol{\Psi}}_{\phi}^{NKOE}$, plays a central role in determining the overall effect of tariffs on inflation. Intuitively, if a given sector is central to production—either because it is widely used across industries (e.g., steel and aluminum) or due to its downstream importance (e.g., semiconductor chips)—it will carry significant weight in the standard Leontief inverse. If this sector also exhibits highly flexible (or rigid) prices—corresponding to a vertical (or horizontal) supply curve with fixed quantity (or highly elastic supply)—and is located in a country with relatively loose (or tight) monetary policy, the inflationary impact of a tariff on that sector will be amplified (or muted) by $\tilde{\boldsymbol{\Psi}}_{\phi}^{NKOE}$. Our analytical solution enables a decomposition of inflationary effects that maps directly onto the global input–output matrix and the model's structural parameters. This decomposition functions as a model-based ex-ante sufficient statistic and provides a tractable tool for counterfactual policy analysis.

The open-economy setup follows standard assumptions in the literature, in which households can trade in either local-currency- or USD-denominated bonds, subject to portfolio adjustment costs that ensure a unique steady-state level of debt. We present a five-equation representation of the model that integrates a standard NK core with production network structures and open-economy features. Nominal rigidity is introduced via Rotemberg price adjustment costs, generating realistic macroeconomic dynamics. In this setup, the output of any country–sector can be used either for final household consumption or as an intermediate input in production, giving rise to rich input–output linkages across countries and sectors. The five vector equations derived from the linearized model are as follows: (i) the New Keynesian IS (NKIS) equation; (ii) the New Keynesian Phillips Curve (NKPC) for producer prices; (iii) a definition of the consumption price vector, which deviates from producer prices due to exchange rate movements and tariff distortions; (iv) an Uncovered Interest Parity (UIP) condition that nests international arbitrage conditions; and (v) an equation of motion for external debt, which also incorporates the market-clearing condition. Together, these equations characterize the equilibrium and nest a broad class of NKOE models.[1]

Our aim is to fully capture the role of global input–output (I–O) linkages, allowing them to "speak" both in theory and in the data. This is particularly important in light of the 2025 tariffs, which are designed to affect both domestic and international supply chains. A standard NKOE model that focuses solely on demand reallocation would miss a major supply shock and its inflationary consequences. While many existing models distinguish between final and intermediate goods, or between tradable and non-tradable goods, our framework allows all goods to be used flexibly. We derive closed-form analytical solutions to the linearized model under a set of simplifying assumptions, the most consequential one being elastic labor supply, following the approach of Golosov and Lucas (2007). Under these assumptions, we obtain two distinct solutions: one in which monetary policy fixes nominal demand, and another in which monetary policy follows a Taylor rule.

We discipline the sectoral heterogeneity in price setting using empirical estimates from Nakamura and Steinsson (2008) and the steady state network using the OECD's Inter-Country Input–Output (ICIO) tables, imposing no *a priori* assumptions on whether a good is purely final or tradable. This modeling flexibility ensures that our quantitative results are not driven solely by the overall share of material inputs in marginal costs, as is often the case in conventional NKOE models. Instead, this relationship arises endogenously from the global I–O structure, which in turn allows our framework to nest other models as special cases.

Having established theoretical results using the linearized version of our model, we proceed to solve the fully non-linear model and conduct four quantitative exercises: (i) validation of the model using the U.S. tariffs on China imposed in 2018; (ii) analysis of the impact of U.S. tariffs announced in April 2025 targeting Europe (20%), China (34%), Mexico (25%),

[1] For example, a model without intermediate inputs—where tariffs affect only demand—can be represented by collapsing the input–output matrix $\mathbf{\Omega}$. Likewise, a model with a single imported intermediate input and a final consumption good corresponds to a structure in which the columns of $\mathbf{\Omega}$ associated with final goods are zero vectors.

Canada (25%), and the rest of the world (10%), under the assumption of no retaliation; (iii) an extension of case (ii) to allow for retaliation, resulting in a full-scale trade war between the U.S. and the rest of the world; and (iv) examination of U.S. "tariff threats in reverse," where a previously announced tariff—eliciting threats of retaliation—is later withdrawn. In the theoretical model with linearized solutions, we assume one-time transitory tariff shocks. In contrast, the fully non-linear quantitative model applies near-permanent shocks, modeled as autoregressive processes with a persistence coefficient of 0.95. This combined approach allows us to quantify the impact of tariff threats—which are inherently transitory—by tracing their transmission through inflation and exchange rate expectations channels.

Consistent with the existing literature, we find that the 2018 U.S. tariffs on China had a muted impact on U.S. inflation. The model estimates an inflation impact of 0.07 percentage points, which is in line with the static estimate of Barbiero and Stein (2025), who find that 2018 tariffs contributed approximately 0.1–0.2 percentage points to PCE inflation. Our model also predicts a 4% appreciation of the U.S. dollar (USD) against the Chinese yuan, aligning with the observed 5.6% dollar appreciation between June 2018 and December 2018. The predicted output loss of 0.2% is also consistent with Fajgelbaum et al. (2020), who estimate combined producer and consumer losses totaling 0.4% of U.S. GDP.

Having completed a road test of the model on the 2018 tariffs, we next analyze the impact of the tariffs announced on April 2nd, 2025–on the so-called "Liberation Day"–targeting Europe, China, Canada, Mexico, and the rest of the world. In the absence of retaliation, we find that U.S. real GDP declines by 0.8%, with consumption falling by 1.0% on impact. Inflation rises by 0.5 percentage points, prompting a 0.5 percentage point increase in nominal interest rates, while real wages decline by 2.7%. The U.S. trade-weighted nominal effective exchange rate (NEER) appreciates by 10%. Mexico and Canada experience sharper contractions due to falling external demand, with GDP declining by 0.9% and 0.3%, respectively, and real wages falling by 2.8% and 1.2%. China's GDP contracts modestly (–0.1%), as a substantial bilateral exchange rate depreciation vis-a-vis dollar of 8.9% helps cushion the trade shock. Interestingly, euro area slightly expands in this case, an increase in GDP of 0.09%.

Next, we examine how these estimates change under a full-fledged trade war in which all targeted countries retaliate against the U.S. Under the global trade war, we find that U.S. real GDP decline doubles as GDP contracts by 1.6%, with inflation rising by 0.8 percentage points and the U.S. NEER appreciating by 4.8%. Canada and Mexico experience the sharpest contractions, with GDP falling by 0.6% and 1.4%, respectively, and inflation increasing by 3.0 and 2.6 percentage points. The euro area contracts by 0.3%, while China and the rest of the world experience smaller declines of 0.1%. Inflationary effects are moderate across most regions: China's inflation rises by 0.5 percentage points, and euro area inflation by

0.2 percentage points. U.S. net exports increase by only 0.5% of steady-state GDP. Given the highly disruptive nature of the tariffs, this modest improvement in the trade balance confirms the intuition that, in a multi-country NKOE setting, tariffs cannot meaningfully reduce the overall trade deficit—consistent with the argument in (Obstfeld, 2025).

Last but not least, we investigate the effects of "tariff threats" deployed for geoeconomic purposes—scenarios in which the U.S. announces tariffs today, prompting trading partners to pledge retaliation in the future, but all tariffs are subsequently withdrawn. This "threat" shock highlights the role of the exchange rate as a forward-looking variable particularly transparent. In a perfect foresight setting, when tariffs are announced today and reversed tomorrow through a subsequent announcement, agents optimize based on the entire sequence of announcements. In the case of transitory tariffs—as explored in our analytical solution—the exchange rate response is muted due to production complementarity and home bias, which limit expenditure switching. However, when the threat of permanent tariffs leads to anticipation of retaliation and a full trade war, the exchange rate immediately adjusts to front-load the anticipated change in consumption behavior. In this scenario, the U.S. NEER appreciates by 4.1% on impact—even in the absence of a contemporaneous change in monetary policy. Real GDP and consumption fall by 0.7% and 0.3%, respectively, while inflation declines by 0.6 percentage points, resulting in deflation. These outcomes are driven primarily by the expectations channel: agents "price in" a future in which the U.S.—a net importer relative to the rest of the world—imports fewer foreign goods. Even before the mechanical price effects of actual tariffs materialize, anticipated trade distortions cause demand to contract, generating deflation on impact. Thus, while actual tariffs shift quantities by affecting supply chains and relative prices, reversed "tariff threats" generate limited quantity adjustments but sizable movements in prices, operating primarily through the expectations channel. A large depreciation of the dollar in the quarter after the threat is reminiscent of exchange rate overshooting and due to expectations and realization of tariffs not taking place. We also show that increased uncertainty surrounding the imposition of tariffs can lead to a depreciation of dollar; we model these uncertainty shocks as UIP wedges imposed at the same time as tariffs.

Overall, our results imply that the inflationary impact of tariffs can be muted, while the effects on output and unemployment can be substantial in the presence of input–output linkages, country–sector heterogeneity in price stickiness, and open-economy channels. NKOE models that do not incorporate full global I–O linkages may systematically overestimate inflation and underestimate the real costs of tariffs, such as decline in employment. NKOE models that focus exclusively on the demand-side effects may suggest that monetary policy should be expansionary in response to falling demand. However, when tariffs exert a significant inflationary impact via their supply shock aspect, the optimal monetary policy response may instead be contractionary. Our results therefore emphasize the need for models that account for the full general equilibrium effects of tariffs—including network propagation and sectoral heterogeneity—when designing and evaluating trade and monetary policy.

Related Literature

In New Keynesian models, changes in monetary policy and exchange rate regimes have non-trivial effects on real variables (e.g., Gali and Monacelli, 2005). As a result, monetary and exchange rate policies serve as both potential stabilization tools and sources of economic fluctuations, particularly in two-country global general equilibrium frameworks where each country's monetary policy responds endogenously to external shocks (e.g., Clarida et al., 2002). Our paper relates to this literature as we explore the interaction between trade policy and monetary policy.

A key contribution of our paper is to generalize and nest the findings of recent work that studies optimal policies as Monacelli (2025), Bergin and Corsetti (2023), and Bianchi and Coulibaly (2025) and extend their results with the supply shock aspect of tariffs in a setting with global networks and sectoral heterogeneity in price rigidities. In these models, tariffs primarily act as demand shocks, influencing consumer prices but not affecting marginal costs or producer price inflation. If tariffs only raise the prices consumers pay without impacting firms' cost structures, they function as an additional force interacting with the exchange rate in the presence of sticky prices. Consequently, their overall effect through the cost channel remains limited. Since they do not act as supply shocks, they do not directly affect production costs or firms' price-setting behavior, causing CPI and PPI to move in opposite directions. This raises the question of whether monetary policy should target CPI or PPI. Additionally, these models often predict a small short-run boost to output following a tariff increase. In contrast, our model demonstrates that if tariffs are large and the supply-side effects dominate, they can have a significantly contractionary impact on output.

We are closely related to recent studies that integrate production networks into the New Keynesian framework in an international setting. One key related work is Cuba-Borda et al. (2025), which presents an empirical examination of trade distortions. While Cuba-Borda et al. (2025) constructs a numerical model to motivate empirical work, our approach provides analytical results that clarify the key mechanisms driving tariff-induced distortions. We are also connected to Ho et al. (2022), who develop a global NK model. Their analytical solution relies on a real rate rule that fixes the path of consumption. In contrast, our model offers a richer analytical characterization of tariff transmission by emphasizing the critical role of endogenous networks and monetary policy in shaping macroeconomic outcomes. Unlike models that impose fixed relative and aggregate consumption paths, our framework allows both demand and policy to respond endogenously, capturing the dynamic interactions between tariffs, exchange rates, and monetary policy.

Our paper also contributes to the rapidly expanding literature on reconstructing macroeconomic aggregates from granular microeconomic components, both theoretically and empirically. Specifically, our model is inspired by Baqaee and Farhi (2024), which examines the effects of tariff shocks in an international production network setting over two periods. Our contribution over their work is to formulate a dynamic open economy NK model, which allows us to focus on short-run inflation-output trade off with exchange rate adjustment and the role of inflation expectations, together with the long-run impact of tariffs, providing the analytical NKOE Leontief inverse. This helps us to decompose the dynamic general equilibrium response to tariff shocks. Other two-period closed and open economy network models are Baqaee and Farhi (2022), who incorporate both supply and demand shocks in a closed-economy context, Di Giovanni et al. (2023) who extends this model to an open-economy setting, and Silva (2024) who explores the interaction between the CPI and production networks in small open economies.

Building on Leontief (1953), there has been a dormant literature to model input–output networks dynamically to capture capital accumulation and investment channels. Long and Plosser (1983) integrated the input–output structure into a real business cycle (RBC) framework, laying the foundation for subsequent work on sectoral shock propagation. Building on this approach, Foerster et al. (2011), Atalay (2017), Foerster et al. (2022), and Vom Lehn and Winberry (2022) apply RBC models to study how shocks propagate through production networks. Huo et al. (2025) further relate international GDP comovements to global production linkages. Our paper is also related to the literature on the propagation of shocks and distortions in static input–output networks, such as Acemoglu et al. (2012); Atalay (2017); Liu (2019); Bigio and La'o (2020); La'O and Tahbaz-Salehi (2022).

The international trade literature has traditionally focused on static real models that analyze the welfare implications of trade costs (Arkolakis et al., 2012; Costinot and Rodríguez-Clare, 2014). Starting with Caliendo and Parro (2015), implications of input-output networks have been studied extensively in the trade literature (see Bernard and Moxnes, 2018; Johnson, 2018, for on overview), together with the global value chains (see Antrás and Chor, 2022, for review). These models provide valuable insights into comparative statics but often abstract from short-run dynamics, expectations, and the role of monetary policy in shaping the transmission of trade shocks. Large-scale computable general equilibrium (CGE) models, such as GTAP (Corong et al., 2017) and G-cubed (McKibbin and Wilcoxen, 2013), partially address these limitations and have been used to study trade policy changes. However, the complexity and scale of these models turn them into a black box, making it difficult to interpret the key economic mechanisms driving their results. Our paper builds on the existing trade literature by incorporating forward-looking behavior and dynamic adjustment while maintaining analytical tractability.

There is also an extensive empirical literature that utilizes detailed micro data to demonstrate a significant pass-through of tariffs into consumer prices, regardless of whether the tariff is imposed on exports or imports (e.g., Amiti et al., 2019; Fajgelbaum et al., 2020; Cavallo et al., 2021; Cox, 2021; Fajgelbaum and Khandelwal, 2022). Our theoretical and quantitative results provide a structural explanation for these reduced form findings, offering novel insights into the mechanisms through which tariffs propagate into consumer prices.

Roadmap. The remainder of this paper is organized as follows. Section 2 outlines our baseline New Keynesian open-economy model, detailing how we incorporate international production networks, nominal rigidity, and open-economy features. Section 3 provides analytical solutions that illustrate how tariffs operate through both demand and supply channels, decomposing their impact. To enhance transparency, we first solve the model under fixed nominal demand before extending the analysis to a Taylor Rule framework. Section 4 presents our quantitative results, applying the model to four key cases: the 2018 U.S.–China trade war, the tariffs announced early in 2025, the full-scale trade war based on April 2025, the "Liberation-Day" announcement and reversed "tariff- threats" done for geoeconomic reasons. Finally, Section 5 concludes.

2 Modeling Framework

We develop a multi-country New Keynesian model that incorporates nominal rigidities via Rotemberg costs, standard open-economy features such as portfolio adjustment costs and trade costs, and a production network.

Households optimize intertemporally, allocating consumption and labor supply while facing portfolio adjustment costs when holding foreign bonds. The production side follows a nested CES structure, with goods classified by sector and origin, and firms producing using labor and intermediate inputs. Prices are set in the producer's currency (PCP) and are subject to revenue-neutral tariffs. Monetary policy follows a Taylor rule, and exchange rates are fully endogenous in the model. Endogenous deviations from Uncovered Interest Parity (UIP) arise due to portfolio adjustment costs; as a country's real USD debt increases, the effective interest rate it pays also rises. The model provides a unified framework for analyzing macroeconomic dynamics in an interconnected global network economy.

2.1 Intertemporal problem.

The household in country n maximizes the present value of lifetime utility:

$$\max_{\{C_{n,t}, L_{n,t}, B_{n,t}^{US}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta_n^t \left[\frac{C_{n,t}^{1-\sigma}}{1-\sigma} - \chi \frac{L_{n,t}^{1+\gamma}}{1+\gamma} \right]$$

subject to

$$\begin{aligned} P_{n,t}C_{n,t} + T_{n,t} - B_{n,t} - \mathcal{E}_{n,t}^{US} B_{n,t}^{US} + \mathcal{E}_{n,t}^{US} \psi(B_{n,t}^{US}/P_{n,t}^{US}) \leq \\ W_{n,t}L_{n,t} + \sum_i \Pi_{ni,t} - (1+i_{n,t-1})B_{n,t-1} - \mathcal{E}_{n,t}^{US}(1+i_{n,t-1}^{US})B_{n,t-1}^{US} \end{aligned}$$

where $P_{n,t}$ is the price of the consumption bundle at time t, and $\mathcal{E}_{n,t}^{US}$ is the exchange rate between country n and the US. An increase in $\mathcal{E}_{n,t}^{US}$ implies a depreciation of the local currency relative to the US dollar. $W_{n,t}$ is the wage in country n at time t, $L_{n,t}$ is the quantity of labor supplied in country n, $i_{n,t}$ is the nominal interest rate in local currency, and $i_{US,t}$ is the interest rate on the US bond, where bonds are treated as liabilities. The term $\psi(B_{n,t}^{US}/P_{n,t}^{US})$ represents a stationarity-inducing portfolio adjustment cost that ensures a unique steady-state level of real debt (i.e., debt denominated in USD, deflated by the US consumer price level). Taxes and transfers are denoted by $T_{n,t}$. In our model, tariffs are revenue-neutral; since there is a lump-sum rebate through $T_{n,t}$, tariff revenues and costs do not affect the budget constraint.

Maximizing the household's lifetime utility subject to the present and future budget constraints yields the following standard first-order conditions (see Appendix A.1):

$$1 = \beta_n E_t \left[\left(\frac{C_{n,t+1}}{C_{n,t}} \right)^{-\sigma} \frac{P_{n,t}}{P_{n,t+1}} (1+i_{n,t}) \right] \forall n \in N, \forall t \qquad \text{(Euler Equation)}, \tag{1}$$

$$\frac{1+i_{n,t}}{1+i_{n,t}^{US}} = E_t \left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}} \right] \frac{1}{1-\psi'(B_{n,t}^{US})} \qquad \text{(UIP) } n \in N-1. \tag{2}$$

The domestic bond is in net zero supply everywhere, and all countries save or dissave using US bonds. In addition to the UIP condition, the arbitrage condition ensures that a country's bilateral exchange rates remain consistent with its exchange rates against the US. Finally, for completeness of notation, we define a country's exchange rate with itself.

$$\mathcal{E}_{n,m,t} = \frac{\mathcal{E}^{US}_{n,t}}{\mathcal{E}^{US}_{m,t}} \ \forall n \neq m \ \& \ m \neq US \ n, m \in N \tag{3}$$

$$\mathcal{E}_{n,n,t} = 1 \ \forall n \in N \tag{4}$$

We have $N \times N$ exchange rates, and along with the UIP condition, these two conditions uniquely determine the exchange rate.

2.2 Intratemporal problem.

We now turn to the household's intratemporal problem. First, we introduce the consumption choices and labor supply decisions. Then, we turn to the production side. The first part of the intratemporal problem is the standard labor-consumption tradeoff that determines labor supply:

$$\frac{W_{n,t}}{P_{n,t}} = \chi L^{\gamma}_{n,t} C^{\sigma}_{n,t} \ \forall n \in N, \forall t \tag{5}$$

Determining the intratemporal breakdown of consumption involves a nested CES structure. Outputs from different countries are first bundled into a country-sector consumption bundle, which is then aggregated into a country good:

$$C_{n,t} = \left[\sum_{i \in J} \Xi_{n,i}^{\frac{1}{\sigma_h}} C_{n,i,t}^{\frac{\sigma_h - 1}{\sigma_h}} \right]^{\frac{\sigma_h}{\sigma_h - 1}}, \tag{6}$$

where $C_{n,i,t}$ is country n's consumption of industry bundle i, and $\Xi_{n,i}$ is the weight of bundle i. This bundle is then a combination of good i procured by country n from countries $m \in N$ globally:

$$C_{n,i,t} = \left[\sum_{m \in N} \Xi_{n,i,mi}^{\frac{1}{\sigma_l^i}} C_{n,i,mi,t}^{\frac{\sigma_l^i - 1}{\sigma_l^i}} \right]^{\frac{\sigma_l^i}{\sigma_l^i - 1}}, \tag{7}$$

where $\Xi_{n,i,mi}$ is the weight of country m's good in this bundle. We can then express the relevant price levels in line with the CES structure:

$$P^c_{n,t} = \left[\sum_{i \in J} \Xi_{n,i} (P^c_{n,i,t})^{1-\sigma_h} \right]^{\frac{1}{1-\sigma_h}}$$

$$P^c_{n,i,t} = \left[\sum_{m \in N} \Xi_{n,i,mi} P^{1-\sigma^i_l}_{n,mi,t} \right]^{\frac{1}{1-\sigma^i_l}}$$

where $P^c_{n,i,t}$ is the local currency consumption price of the aggregated good basket i in country n at time t.

We assume that prices are set in the producer's currency and then converted to the consumer's currency using the exchange rate under the producer currency pricing (PCP) assumption:

$$P_{n,mi,t} = \mathcal{E}_{n,m,t}(1 + \tau_{n,mi,t}) P_{mi,t} \tag{8}$$

where $\mathcal{E}_{n,m,t}$ is the bilateral exchange rate.

To complete the specification of demand on the household side, we need to define the relative demand conditions given the nested CES structure. Consumers choose:

$$C_{n,i,t} = \Xi_{n,i} \left(\frac{P^c_{n,i,t}}{P_{n,t}} \right)^{-\sigma_h} C_{n,t} \tag{9}$$

$$C_{n,mi,t} = \Xi_{n,i,mi} \left(\frac{P_{n,mi,t}}{P^c_{n,i,t}} \right)^{-\sigma^i_l} C_{n,i,t} \tag{10}$$

2.3 Production

Having defined the household's side, we now turn to the production side of the economy. Output in country n, sector i, at time t follows a CES production function:

$$Y_{ni,t} = A_{ni,t} \left[\alpha^{1/\theta}_{ni} L^{\frac{\theta-1}{\theta}}_{ni,t} + (1 - \alpha_{ni})^{1/\theta} (X_{ni,t})^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \forall n \in N, \forall i \in J, \forall m \in N, \forall j \in J \tag{11}$$

All firms within a given country-sector combination are assumed to be identical, and each firm solves the following marginal cost minimization problem:

$$MC_{ni,t} = \min_{\{X_{ni,j,t}, L_{ni,t}\}} W_t L_{ni,t} + P^p_{ni,t} X_{ni,t} \quad \text{s.t.} \quad Y_{ni,t} = 1.$$

As a firm faces this problem, it chooses labor and the quantities of the intermediate good specific to the producing industry in the given country. This intermediate good bundle is constructed as follows. Intermediate goods from different countries are first bundled into a country-industry-good bundle. This bundle and the relevant relative demand condition are defined below:

$$X_{ni,j,t} = \left[\sum_{m \in N} \Omega_{ni,j,mj}^{\frac{1}{\theta_l^j}} X_{ni,mj,t}^{\frac{\theta_l^j - 1}{\theta_l^j}} \right]^{\frac{\theta_l^j}{\theta_l^j - 1}} \tag{12}$$

$$X_{ni,mj,t} = \Omega_{ni,j,mj} \left(\frac{P_{n,mj,t}}{P^p_{ni,j,t}} \right)^{-\theta_l^j} X_{ni,j,t} \tag{13}$$

where $P^p_{ni,j,t}$ is the average price of j for producer sector i in country n. Analogously, the intermediate bundle is constructed as follows:

$$\frac{X_{ni,j,t}}{X_{ni,t}} = \Omega_{ni,j} \left(\frac{P_{ni,j,t}}{P^p_{ni,t}} \right)^{-\theta_h} \forall\ j \in J \tag{14}$$

$$X_{ni,t} = \left[\sum_{j \in J} \Omega_{ni,j}^{\frac{1}{\theta_h}} X_{ni,j,t}^{\frac{\theta_h - 1}{\theta_h}} \right]^{\frac{\theta_h}{\theta_h - 1}} \tag{15}$$

As we derive in detail in Appendix A.2, given the setup and definitions above, the firm's problem yields the following equilibrium conditions:

$$\frac{X_{ni,t}}{L_{ni,t}} = \frac{(1-\alpha_{ni})}{\alpha_{ni}} \left(\frac{W_t}{P^p_{ni,t}} \right)^{\theta} \tag{16}$$

$$MC_{ni,t} = \frac{1}{A_{ni,t}} \left[\alpha_{ni} W_t^{1-\theta} + (1-\alpha_{ni}) \left(\sum_j \Omega_{ni,j} P_{ni,j,t}^{1-\theta_h} \right)^{\frac{1-\theta}{1-\theta_h}} \right]^{\frac{1}{1-\theta}} \tag{17}$$

Within each country-sector, there is an infinite continuum of identical firms. A representative firm f in sector i of country n solves the following problem under the Rotemberg setup:

$$P^f_{ni,t} = \arg\max_{P^f_{ni,t}} \mathbb{E}_t \left[\sum_{T=t}^{\infty} \mathrm{SDF}_{t,T} \left[Y^f_{ni,T}(P^f_{ni,T}) \left(P^f_{ni,T} - MC_{ni,T} \right) - \frac{\delta_{ni}}{2} \left(\frac{P^f_{ni,T}}{P^f_{ni,T-1}} - 1 \right)^2 Y_{ni,T} P_{ni,T} \right] \right]$$

where a bundler aggregates the sectoral output into a CES bundle such that the demand function is $Y^f_{ni,t}(P^f_{ni,t}) = \left(\frac{P^f_{ni,t}}{P_{ni,t}} \right)^{-\theta_r} Y_{ni,t}$. As we show in Appendix A.2.1, this problem yields the following equilibrium condition:

$$\left(\Pi_{ni,t} - 1 \right) \Pi_{ni,t} = \frac{\theta_r}{\delta_{ni}} \left(\frac{MC_{ni,t}}{P_{ni,t}} - \frac{\theta_r - 1}{\theta_r} \right) + \beta_n \mathbb{E}_t \left[\left(\Pi_{ni,t+1} - 1 \right) \Pi_{ni,t+1} \right] \tag{18}$$

Equation (18) constitutes a country- and sector-specific forward-looking New Keynesian Phillips Curve, expressed in terms of nominal marginal cost deflated by the sector's producer price. As $\delta_{ni} \to 0$, prices become more flexible, leading to $\Pi_{n,t} = 1$ and $\frac{MC_{ni,t}}{P_{ni,t}} = \frac{\theta_r - 1}{\theta_r}$, which corresponds to the general pricing equation under monopolistic competition.

2.4 Balance of Payments and NIIP

We track the evolution of each country's net international investment position (NIIP) as follows:

$$\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}}\left(\frac{P_{n,mj,t}}{1+\tau_{n,mi,t}}C_{n,mj,t}\right) + \sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}}\left(\frac{P_{n,mj,t}}{1+\tau_{n,mi,t}}X_{ni,mj,t}\right) + \mathcal{E}_{n,t}(1+i^{US}_{n,t-1})B^{US}_{n,t-1}$$
$$+\, \mathcal{E}_{n,t}\psi(B^{US}_{n,t}/P^{US}_{n,t}) = \sum_{i\in\mathcal{J}}(P_{ni,t}Y_{ni,t}) + \mathcal{E}_{n,t}B^{US}_{n,t} \quad \forall n \in N-1 \tag{19}$$

where we account for the fact that tariffs are modeled as revenue-neutral by dividing relevant prices by $(1+\tau_{n,mi,t})$, since end-user prices reflect the impact of tariffs just as they do the impact of exchange rates. This will play an important role when we switch to vector notation in Section 3.

Given market-clearing conditions and budget constraints, one country's budget constraint is redundant as an equilibrium condition. Thus, we omit that of the first country, which corresponds to the US in our model. However, we still need to ensure that the market for USD bonds is closed:

$$B^{US}_t = \sum_{m}^{N-1} B^{US}_{m,t} \tag{20}$$

2.5 Definitions, Market Clearing, Policy and Equilibrium

We assume that all goods markets clear. Goods can be used as final (consumption) goods and as intermediate inputs in all countries. Therefore, we write the goods market-clearing condition for country-sector ni at time t as:

$$Y_{ni,t} = \sum_{n\in\mathcal{N}}(C_{m,ni,t}) + \sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}}(X_{mj,ni,t}), \tag{21}$$

where country m is the consuming country and n is the producing country.

To close the model, we need to specify the market-clearing condition for labor, define aggregate inflation, and specify policy. Policy in each country follows a standard Taylor rule.

$$L_{n,t} = \sum_{i \in J} L_{ni,t} \tag{22}$$

$$\Pi_{n,t} = \frac{P_{n,t}}{P_{n,t-1}} \quad \forall n \in N \tag{23}$$

$$1 + i_{n,t} = (\Pi_{n,t})^{\phi_\pi} e^{\hat{M}_{n,t}} \quad \forall n \in N \tag{24}$$

where $\hat{M}_{n,t}$ is a policy shock.

Definition 1. A non-linear competitive equilibrium for the model is a sequence of 11 endogenous variables $\{C_{nt}, C_{ni,t}, C_{n,mj,t}, X_{ni,mj,t}, X_{ni,j,t}, X_{ni,t}, Y_{ni,t}, L_{ni,t}, L_{n,t}, MC_{ni,t}, B^{US}_{n,t}\}_{t=0}^{\infty}$ and 11 prices $\{P_{nt}, P_{ni,t}, P^c_{ni,t}, P^p_{ni,t}, P^p_{ni,j,t}, P_{n,mi,t}, \Pi_{n,t}, \Pi_{ni,t}, \mathcal{E}_{n,t}, i_{n,t}, W_{n,t}\}_{t=0}^{\infty}$ given exogenous processes $\{\tau_t, A_{ni,t}, \hat{M}_{n,t}\}_{t=0}^{\infty}$ such that equations (1)-(24) hold for all countries and time periods.

3 Linearized Model

We can linearize the 24 equations above to define an approximated equilibrium and use the method of undetermined coefficients to solve the model analytically, expressing all endogenous variables as functions of lagged variables and exogenous shocks (tariffs). With this analytical solution, we can derive some of our theoretical results.

Definition 2. A linearized competitive equilibrium for the model is a sequence of 11 endogenous variables $\{\hat{C}_{n,t}, \hat{C}_{ni,t}, \hat{C}_{n,mj,t}, \hat{X}_{ni,mj,t}, \hat{X}_{ni,j,t}, \hat{X}_{ni,t}, \hat{Y}_{ni,t}, \hat{L}_{ni,t}, \hat{L}_{n,t}, \widehat{MC}_{ni,t}, \hat{B}^{US}_{n,t}\}_{t=0}^{\infty}$ and 11 prices $\{\hat{P}_{nt}, \hat{P}_{ni,t}, \hat{P}^c_{ni,t}, \hat{P}^p_{ni,t}, \hat{P}^p_{ni,j,t}, \hat{P}_{n,mi,t}, \hat{\Pi}_{n,t}, \hat{\Pi}_{ni,t}, \hat{\mathcal{E}}_{n,t}, \hat{i}_{n,t}, \hat{W}_{n,t}\}_{t=0}^{\infty}$ given exogenous processes $\{\hat{\tau}_t, \hat{A}_{ni,t}, \hat{M}_{n,t}\}_{t=0}^{\infty}$ such that equations (52)-(73) hold for all countries and time periods.

Solving the model analytically requires making some simplifying assumptions. The first simplifying assumption involves adopting Golosov and Lucas (2007) preferences. Relatedly, we set $\chi = \sigma = 1$ and the disutility of labor $\gamma = 0$, making labor infinitely elastic. Second, while we assume that portfolio adjustment costs continue to ensure the uniqueness of the steady-state level of debt in the model, numerically we assume $\psi(B^{US}_{n,t}) \to 0$.[2] Real-world debt data exhibits high persistence, and when we calibrate portfolio adjustment costs accordingly, quantitative simulations show that the contribution of this term is small. Third,

[2]Portfolio adjustment costs serve as our stationarity-inducing device. In our analytical work, when we forward forward-looking variables, we assume that portfolio adjustment costs, along with a sufficiently high ϕ_π, ensure that in the long run, all real variables return to steady-state levels in response to transitory shocks.

in the analytical model, for narrative ease, we assume that all shocks other than tariffs are set to zero (e.g., $A_{ni,t} = 0 \ \forall n, i, t$). Finally, we introduce generalized elasticity and weight terms that directly link the lowest-level bundles to the highest-level aggregates, such as:[3]

$$\hat{C}_{nt} = \sum_{m \in N} \sum_{i \in J} \Xi_{n,mi} \hat{C}_{n,mi,t} = 0$$
$$\hat{C}_{n,mi,t} = -\sigma_l^i \left(\hat{P}^p_{mi,t} + \hat{\mathcal{E}}_{n,m,t} + \tau_{n,mi,t} - \hat{P}^c_{ni,t} \right)$$

3.1 Vector and Matrix Notation

Given the number of countries and industries involved, our equilibrium conditions are difficult to solve and analyze in scalar form. The most important aspect of the matrix notation pertains to deriving the New Keynesian Phillips Curve. To that end, let us consider the linearized producer price inflation equation:

$$\pi^p_{ni,t} = \frac{\theta_l}{\delta_{ni}} \left(\alpha_{ni} \hat{W}_t + \sum_{m \in N} \sum_{j \in J} \Omega_{ni,mj} \underbrace{\left(\hat{P}^p_{ni,t} + \hat{\mathcal{E}}_{n,m,t} + \tau_{n,mj,t} \right)}_{\widehat{MC}_{ni,t}} - \hat{P}^p_{ni,t} \right) + \beta \mathbb{E}_t \pi^p_{ni,t+1}$$

This can be expressed in vector and matrix notation as follows:

$$\begin{aligned} \underbrace{\boldsymbol{\pi}^P_t}_{NJ\times 1} = \underbrace{\boldsymbol{\Lambda}}_{NJ\times NJ} \Bigg(& \underbrace{\boldsymbol{\alpha}}_{NJ\times N} \underbrace{\hat{\boldsymbol{W}}_t}_{N\times 1} + \underbrace{(\boldsymbol{\Omega} - \boldsymbol{I})}_{NJ\times NJ} \underbrace{\hat{\boldsymbol{P}}^P_t}_{NJ\times 1} + [\underbrace{\boldsymbol{\Omega}}_{NJ\times NJ} \odot \underbrace{\hat{\boldsymbol{\mathcal{E}}}_t}_{NJ\times NJ}] \underbrace{\boldsymbol{1}}_{NJ\times 1} \\ & + [\underbrace{\boldsymbol{\Omega}}_{NJ\times NJ} \odot \underbrace{\hat{\boldsymbol{\tau}}_t}_{NJ\times NJ}] \underbrace{\boldsymbol{1}}_{NJ\times 1} \Bigg) + \boldsymbol{\beta} \mathbb{E}_t \underbrace{\boldsymbol{\pi}^P_{t+1}}_{NJ\times 1} \end{aligned} \tag{25}$$

where, with some slight abuse of notation, we define the tariff matrix as $\hat{\boldsymbol{\tau}}_{ni,mj,t} \equiv \tau_{n,mi,t}$, the exchange rate matrix as $\hat{\boldsymbol{\mathcal{E}}}_{ni,mj,t} = \hat{\mathcal{E}}_{n,m,t}$ and the diagonal matrix of discount rates as $\boldsymbol{\beta}_{nn} \equiv \beta_n$.

Thus, keeping in mind the labor-leisure tradeoff and using the fact that the price level

[3] To the first order, bundles presented in Sections 2.2 and 2.3 can be directly linked to the goods that form them. We can write these relations as:

$$\Xi_{n,mi} = \Xi_{n,i} \ \Xi_{n,i,mi},$$
$$\Omega_{ni,mj} = (1 - \alpha_{ni}) \ \Omega_{ni,j} \ \Omega_{ni,j,mj},$$

at time t is the past price level plus inflation, we can express producer prices in levels as:

$$\hat{\boldsymbol{P}}_t^P = \underbrace{(\boldsymbol{I}(1+\beta) + \boldsymbol{\Lambda}(\boldsymbol{I}-\boldsymbol{\Omega}))^{-1}}_{\tilde{\boldsymbol{\Psi}}} \left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda} \left(\boldsymbol{\alpha} \underbrace{\left(\hat{\boldsymbol{P}}_t^C + \sigma \hat{\boldsymbol{C}}_t \right)}_{\hat{\boldsymbol{W}}_t} + [\boldsymbol{\Omega} \odot \hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} + [\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1} \right) + \beta \mathbb{E}_t \hat{\boldsymbol{P}}_{t+1}^P \right]$$

where $\tilde{\boldsymbol{\Psi}}$ is a stickiness-adjusted Leontief Inverse.

We can also express the CPI using these matrices. For analytical tractability, we define the $NJ \times 1$ dimensional CPI vector $\boldsymbol{P}_t^C$ such that $\boldsymbol{P}_{mi,t}^C = P_{m,t}^c$. With this, we can write the CPI as:

$$\hat{\boldsymbol{P}}_t^C = \boldsymbol{\Xi} \cdot \hat{\boldsymbol{P}}_t^P + [\boldsymbol{\Xi} \odot \hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} + [\boldsymbol{\Xi} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1},$$

where $\boldsymbol{\Xi}$ is an $NJ \times NJ$ matrix such that $\boldsymbol{\Xi}_{ni,mj} = \Xi_{n,mj}$.

The matrix notation makes our expressions compact, generalizable, and useful for computational work. That said, in equilibrium definitions and macroeconomic interpretation, we additionally use vector notation. To that end, consider $\tilde{\boldsymbol{\mathcal{E}}}_t$ as the $N^2 \times 1$ vector of bilateral exchange rates that determine the entries of the matrix $\hat{\boldsymbol{\mathcal{E}}}_t$. Similarly, let $\tilde{\boldsymbol{\tau}}_t = \text{vec}(\boldsymbol{\tau}_t)$ be the vectorized form of $\boldsymbol{\tau}_t$, where $\tilde{\boldsymbol{\tau}}_t$ is an $N^2J \times 1$ vector. In line with these vector representations, we also use $\mathbf{L}$ to denote loadings (i.e., how the subscript variable loads onto the superscript variable). These expressions compactly describe how vector variables load onto a given equation and serve as partial derivatives.

3.2 Global New Keynesian Representation

With the vector and matrix notation established, the full set of linearized equilibrium conditions in Appendix B can be written in vector form as an equilibrium that satisfies the Blanchard-Kahn stability conditions. We use this representation both for interpretation and to solve the model using the method of undetermined coefficients. This five-equation representation is similar in spirit to the canonical three-equation New Keynesian model, if that model were extended to a context with N open economies, including input-output linkages.

Definition 3. A linearized equilibrium comprises vector sequences $\{\hat{\boldsymbol{C}}_t, \hat{\boldsymbol{P}}_t^P, \hat{\boldsymbol{P}}_t^C, \tilde{\boldsymbol{\mathcal{E}}}_t, \hat{\boldsymbol{V}}_t\}_{t_0}^{\infty}$ for a given sequence of $\{\hat{\boldsymbol{\tau}}_t\}_{t_0}^{\infty}$ and an initial condition for $\hat{\boldsymbol{V}}_0$ such that equations (26)-(30) hold:

$$\textbf{NKIS+TR:} \quad \sigma(\mathbb{E}_t \hat{\boldsymbol{C}}_{t+1} - \hat{\boldsymbol{C}}_t) = \boldsymbol{\Phi}(\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_{t-1}^C) - \mathbb{E}_t(\hat{\boldsymbol{P}}_{t+1}^C - \hat{\boldsymbol{P}}_t^C) \tag{26}$$

$$\textbf{CPI:} \quad \hat{\boldsymbol{P}}_t^C = \boldsymbol{\Xi} \hat{\boldsymbol{P}}_t^P + \tilde{\boldsymbol{L}}_{\mathcal{E}}^C \tilde{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}_{\tau}^C \tilde{\boldsymbol{\tau}}_t \tag{27}$$

$$\textbf{NKPC:} \quad \hat{\boldsymbol{P}}_t^P = \tilde{\boldsymbol{\Psi}} \left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda} \left(\boldsymbol{\alpha} \left(\hat{\boldsymbol{P}}_t^C + \sigma \hat{\boldsymbol{C}}_t \right) + \boldsymbol{L}_{\mathcal{E}}^P \tilde{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}_{\tau}^P \hat{\boldsymbol{\tau}}_t \right) + \beta \mathbb{E}_t \hat{\boldsymbol{P}}_{t+1}^P \right] \tag{28}$$

$$\textbf{UIP+TR:} \qquad \tilde{\mathbf{\Phi}}_1 \mathbb{E}_t \tilde{\boldsymbol{\mathcal{E}}}_{t+1} - \tilde{\mathbf{\Phi}}_2 \tilde{\boldsymbol{\mathcal{E}}}_t = \tilde{\mathbf{\Phi}}_3 (\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_{t-1}^C) \tag{29}$$

$$\textbf{BoP:} \qquad \beta \hat{\boldsymbol{V}}_t = \mathbf{\Gamma}_1 \hat{\boldsymbol{V}}_{t-1} + \mathbf{\Gamma}_2 \hat{\boldsymbol{C}}_t + \mathbf{\Gamma}_3 \hat{\boldsymbol{P}}_t^P + \Gamma_4 \tilde{\boldsymbol{\mathcal{E}}}_t + \Gamma_5 \tilde{\boldsymbol{\tau}}_t \tag{30}$$

where "TR" denotes that the Taylor rule has been substituted in, and $\mathbf{L}$ notation represents loadings (i.e., how the subscript variable loads onto the superscript variable as a linear combination of the entries of the vector variable, as detailed above), which also serve as partial derivatives. In the first and fourth of these equilibrium conditions, the Taylor rule is used to substitute out the nominal interest rate, where the diagonal matrix $\mathbf{\Phi}$ contains the Taylor rule's sensitivity to inflation in the respective countries. For example, in the two-country case, we have $\mathbf{\Phi} = \begin{bmatrix} \phi_\pi & 0 \\ 0 & \phi_\pi^* \end{bmatrix}$. That is, we have $\hat{\boldsymbol{i}}_t = \mathbf{\Phi}(\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_{t-1}^C)$ and the first $N-1$ rows of $\tilde{\mathbf{\Phi}}_3(\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_{t-1}^C)$ load the vector form of interest rate differentials $\hat{\boldsymbol{i}}_t - \hat{i}_t^{US}$ for countries other than the first country in our system, the US.

The first of these equilibrium conditions is the Euler (New Keynesian IS, i.e., NKIS) equation, which is defined in terms of aggregate consumer prices. Intuitively, the impact of tariffs enters the demand side through how tariffs load onto consumer prices.

The second equation defines the consumer price index (CPI). As is typical in network models, the CPI and the producer price index (PPI) differ, with consumer prices being a weighted average of producer prices, exchange rates, and tariffs under our producer currency pricing assumption. Here, $\boldsymbol{L}_{\hat{\mathcal{E}}}^C$ captures, in matrix form, how consumer prices of various goods are exposed to the exchange rate. The scalar analogy would be $(1-\eta)$, where $\eta \in [0,1]$ represents the home bias parameter for consumption. Similarly, $\boldsymbol{L}_\tau^C$ captures the share of goods exposed to tariffs. If one were to momentarily ignore the impact of tariffs on producer prices, their effect via consumer prices would be isomorphic to Euler equation shocks (e.g., discount rate shocks).

The third equation is the New Keynesian Phillips Curve for producer price inflation, defined in levels for convenience in the analytical solution. The impact of the input-output network is captured in the stickiness-adjusted Leontief inverse term $\tilde{\mathbf{\Psi}}$. This term multiplies the diagonal matrix of stickiness parameters, $\mathbf{\Lambda}$, and the matrix of nominal marginal costs. Additionally, $\tilde{\mathbf{\Psi}}$ multiplies both the vector of lagged producer prices, $\hat{\boldsymbol{P}}_{t-1}^P$, and the discounted expectation of future producer prices, $\beta \mathbb{E}_t \hat{\boldsymbol{P}}_{t+1}^P$. In this setup, the exchange rate loads onto nominal marginal costs via the dependence of producers on imported intermediate inputs, which is captured by $\mathbf{L}_{\hat{\mathcal{E}}}^P$. Similarly, tariffs have a direct impact, as they load onto the share of goods exposed to tariffs, captured by $\mathbf{L}_\tau^P$. If not for their additional impact on consumer prices, tariffs τ would be isomorphic to standard supply shocks in the New

Keynesian context.[4]

The fourth equation combines the UIP condition, exchange rate arbitrage conditions, and the definition of a country's exchange rate with itself (i.e., nesting linearized versions of equations (2), (3), and (4)). Here, the $\tilde{\mathbf{\Phi}}$ terms ensure that the ϕ_π terms for each country, along with the arbitrage conditions, are correctly loaded in each row.

The fifth equation combines market clearing for debt with the $N-1$ equations of motion for real debt, capturing the balance of payments as a function of prices, which reflect the terms of trade for each specific country-good variety, and the aggregate consumption vector.[5] This final equation describes how a country's net external position evolves in response to changes in good-specific terms of trade, as well as fluctuations in the interest rate and the balance sheet effect of debt via exchange rates. As such, it nests all the intratemporal relative demand conditions and pricing equations. Through this equation, debt responds to automatic debt dynamics and adjustments in exports following changes in the terms of trade.

This five-equation general representation can nest a broad class of open-economy New Keynesian models. For example, models with a bundle of intermediate inputs and a final good correspond to the case where $\mathbf{\Omega}$ involves $J = 2$, and one of the columns of $\mathbf{\Omega}$ is a column of zeros. This representation is general for N-country New Keynesian models (e.g., Clarida et al., 2002). However, by collapsing the number of countries to one and making the real rate exogenous, it reduces to a small open economy model reminiscent of Gali and Monacelli (2005).

3.3 Inflation Under Fixed Nominal Demand

In order to derive an analytical solution for the equilibrium in Definition 3, it is useful to impose assumptions on aggregate demand to ensure tractability. The first approach we employ is replacing the Taylor rule with the equation: $\hat{P}_t + \hat{C}_t = \hat{M}_t$ which fixes nominal domestic demand. This approach is similar to menu cost models such as Golosov and Lucas (2007); Caratelli and Halperin (2023) and can be microfounded using a cash-in-advance constraint. Since money balances drop from the balance of payments due to the government's

[4] As we discuss in Section 4.6, the impact of tariffs on the supply side can be isomorphic to both productivity shocks and cost-push shocks, depending on the policy choice of the central bank. In a setup where the central bank does not target deviation of output from the pre-tariff steady-state level of output, tariffs can function similar to productivity shocks that change potential output (i.e. the flexible-price equilibrium) as they increase firms' marginal costs. If central banks target deviation of output from pre-tariff levels, then tariffs can act similar to cost-push shocks.

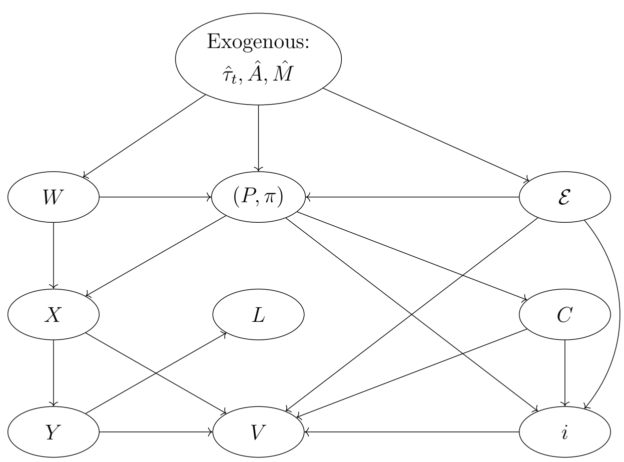
[5] The first $N-1$ rows contain linearized versions of equation (19), while the last row captures the bond market clearing condition given by equation (20). In Appendix E, we derive this equation of motion.

budget constraint, we do not explicitly model money in the utility function.[6] With $\sigma = 1$ and this simplification, we obtain $\hat{W}_{n,t} = \hat{M}_{n,t} = \hat{P}_{n,t} + \hat{C}_{n,t}$.

The economic interpretation is that with an exogenous $\hat{M}_{n,t}$, policy sets the overall aggregate domestic demand stance, similar to earlier generations of models such as Salter-Swan (Swan, 1963; Salter, 1959). In a closed-economy setting, the policy rule would be analogous to nominal GDP targeting. This representation is also similar to real rate rules in the context of HANK models (see, e.g., Kaplan et al., 2018), where it helps to pin down aggregate demand. Here, domestic nominal demand is determined by policy. Explicating the policy response as a state variable has a useful interpretation in this context. First, it allows us to study the impact of tariffs while keeping demand constant (and, as it turns out, also the nominal exchange rate channel). Second, results in this context depend on how hawkish a given country is in responding to inflation relative to other countries. In that sense, defining $\hat{M}_{n,t}$ as an exogenous state variable is helpful. Moreover, many countries do not strictly follow a combination of pure float and inflation targeting. With fiscal policy and foreign exchange intervention, governments can influence domestic demand and the nominal exchange rate. Thus, the abstraction at hand provides a useful framework for analysis.

The Directed Acyclic Graph (DAG) representation in Figure 1 illustrates how wages and the nominal exchange rate can be solved by starting with the policy node, $\hat{M}$. This allows one to derive the price and inflation vectors, which in turn determine all quantities. Using these quantities and prices, the external debt/asset position (and thus the current account) can be determined via the Balance of Payments equation.

Figure 1. Directed Acyclic Graph (DAG) Representation of the Simplified Equilibrium



[6]This approach can also be microfounded by incorporating money in the utility function, which typically introduces some sensitivity of money demand to the interest rate. Cash-in-advance constraints that do not exhibit interest rate sensitivity are often assumed in models without explicit microfounded derivations. However, they can be microfounded by including the term $\chi_M \frac{1+i_{n,t}}{i_{n,t}} \ln\left(\frac{M_{n,t}}{P_{n,t}}\right)$ in the utility function.

3.3.1 Nominal Exchange Rate

With the simplifying assumptions introduced above, the Backus Smith condition can be written and transformed as follows:

$$
\begin{aligned}
E_t \Delta \hat{Q}_{n,m,t+1} &= \sigma \left(E_t \Delta \hat{C}_{n,t+1} - E_t \Delta \hat{C}_{m,t+1} \right) \\
E_t \Delta \hat{\mathcal{E}}_{n,m,t+1} &= E_t \left(\Delta \hat{M}_{n,t+1} - \Delta \hat{M}_{m,t+1} \right) \\
\hat{\mathcal{E}}_{n,m,t} &= \overline{\mathcal{E}}_{n,m} + E_t \left[\sum_{j=0}^{\infty} -\Delta \hat{M}_{n,t+j+1} + \Delta \hat{M}_{m,t+j+1} \right]
\end{aligned}
$$

where $\overline{\mathcal{E}}_{n,m} = \lim_{t\to\infty} \hat{\mathcal{E}}_{n,m,t} \approx 0$ for the type of one-time shocks that we are interested in. To that end, let us assume $M_{n,t+j} = M_{m,t+j} = 0 \; \forall \; j > 0$. Then, we have $\mathcal{E}_{n,m,t} = \hat{M}_{n,t} - \hat{M}_{m,t}$. That is, in the simplified version of the model with fixed nominal demand, nominal demand policy determines the path of nominal exchange rates. The intuitive interpretation of the expression above is that excessively stimulating demand (i.e., printing too much money) leads to depreciation, consistent with models of monetary exchange rate determination.

3.3.2 Producer Price Inflation

Having solved for the nominal exchange rate, we now turn to the Phillips Curve under fixed nominal demand. As detailed in Appendix C.1, in vector and matrix notation, we obtain:[7]

$$
\boldsymbol{\pi}_t^P = \boldsymbol{\Lambda} \Bigg((\boldsymbol{\Omega} - \boldsymbol{I}) \hat{\boldsymbol{P}}_t^P + (\boldsymbol{I} - \boldsymbol{\Omega}) \hat{\boldsymbol{M}}_{\boldsymbol{t}} + [\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t] \mathbf{1} \Bigg) + \beta \mathbb{E}_t \boldsymbol{\pi}_{t+1}^P \tag{31}
$$

The term $(\boldsymbol{I} - \boldsymbol{\Omega})$ in front of $\hat{\boldsymbol{M}}_{\boldsymbol{t}}$ consists of the summation of two components: (i) $\boldsymbol{\alpha}$, which arises from the demand channel via an increase in wages, and (ii) $(\boldsymbol{I} - \boldsymbol{\alpha} - \boldsymbol{\Omega})$, which originates from the exchange rate channel that raises input prices.

An intuitive way to interpret (31) is to first examine the flexible price case, where marginal cost equals price:

$$
\begin{aligned}
\hat{\boldsymbol{\pi}}_t^P &= \underbrace{(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}}_{\text{Leontief Inverse}} \underbrace{(\boldsymbol{I} - \boldsymbol{\Omega})}_{\substack{\text{Policy Impact} \\ \text{via Demand and ER}}} \hat{\boldsymbol{M}}_t + (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} \underbrace{[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t] \mathbf{1}}_{\text{Tariff Incidence}} - \hat{\boldsymbol{P}}_{t-1}^P \\
&= \hat{\boldsymbol{M}}_t + (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} [\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t] \mathbf{1} - \hat{\boldsymbol{P}}_{t-1}^P
\end{aligned} \tag{32}
$$

[7]As described in Appendix C.1, we construct an $NJ \times 1$ dimensional vector $\hat{\boldsymbol{M}}_t$ by stacking each country's nominal demand change such that $\hat{\boldsymbol{M}}_{ni} = \hat{M}_n$.

Equation (32) illustrates the impact on inflation under flexible prices. Nominal domestic demand policy affects producer price inflation through two channels: first, via the demand channel, and second, via the exchange rate channel. Since the labor-leisure tradeoff simplifies to $\hat{W}_t - \hat{P}_t = \hat{C}_t$ under the given parametrization, and since nominal wages depend on $\hat{M}_t$, stimulative demand policy increases labor supply. Through the exchange rate channel, stimulating domestic demand beyond its steady-state level results in depreciation, which raises firms' marginal costs by increasing the price of imported intermediate inputs.

Returning to the Rotemberg pricing case with the forward-looking NKPC in Equation (31), we simplify and define the stickiness-adjusted Leontief inverse for the producer price inflation equation as $\tilde{\boldsymbol{\Psi}}_\pi = [\boldsymbol{I} - \boldsymbol{\Lambda}(\boldsymbol{\Omega} - \boldsymbol{I})]^{-1}$, arriving at the global NKPC for producer price inflation:[8]

$$\boldsymbol{\pi}_t^P = \underbrace{\tilde{\boldsymbol{\Psi}}_\pi \boldsymbol{\Lambda}}_{\substack{\text{Propagation under}\\ \text{stickiness}}} \left[\underbrace{(\boldsymbol{I} - \boldsymbol{\Omega})}_{\substack{\text{Policy impact}\\ \text{via demand and ER}}} \hat{\boldsymbol{M}}_t + \underbrace{[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1}}_{\text{Tariff incidence}} - \underbrace{(\boldsymbol{I} - \boldsymbol{\Omega})\boldsymbol{P}_{t-1}^P}_{\substack{\text{Impact of}\\ \text{lagged prices}}} + \underbrace{\beta \boldsymbol{\Lambda}^{-1} \mathbb{E}_t \boldsymbol{\pi}_{t+1}^P}_{\substack{\text{Forward-looking}\\ \text{behavior}}} \right] \tag{33}$$

Applying the method of undetermined coefficients to (33) and assuming tariff and policy shocks are one-time shocks such that in t+1 they are gone we arrive at Proposition 1.

Proposition 1. *With future shocks set to zero such that (i.e. $\tau_{t+j} = \hat{M}_{t+j} = \hat{M}^*_{t+j} = 0 \ \forall j > 0$) the solution for producer price inflation is that:*

$$\boldsymbol{\pi}_t^P = \tilde{\boldsymbol{\Psi}}^{NKOE} \boldsymbol{\Lambda} (\boldsymbol{I} - \boldsymbol{\Omega}) \hat{\boldsymbol{M}}_t + \tilde{\boldsymbol{\Psi}}^{NKOE} \boldsymbol{\Lambda} [\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1} + (\tilde{\boldsymbol{\Psi}}^{NKOE} - \boldsymbol{I})\boldsymbol{P}_{t-1}^P \tag{34}$$

where $\tilde{\boldsymbol{\Psi}}^{NKOE}$ is the NKOE Leontief inverse in this context. It transforms the stickiness-adjusted Leontief inverse by diagonalizing it and solving a quadratic equation to determine the matrix in front of the lagged vector $\boldsymbol{P}_{t-1}^P$, denoted as $\mathbf{C}_3$. The NKOE Leontief inverse $\tilde{\boldsymbol{\Psi}}^{NKOE}$ is constructed by taking this solution $\boldsymbol{C}_3$ and multiplying it as follows: $\tilde{\boldsymbol{\Psi}}^{NKOE} = \boldsymbol{Q}\boldsymbol{C}_3\boldsymbol{Q}^{-1}$, where $\boldsymbol{Q}$ diagonalizes $\tilde{\boldsymbol{\Psi}}_\pi$. In this formulation, $\tilde{\boldsymbol{\Psi}}^{NKOE}$ also serves as the matrix that multiplies the lagged price vector when producer price inflation is expressed in levels.

Proof. See Appendix C. □

[8] For intuition, in the closed-economy analogy, there is no exchange rate impact, and tariffs would act as a cost-push shock, with $\hat{\boldsymbol{M}}_t$ capturing NGDP.

$$\boldsymbol{\pi}_t^P = \underbrace{\tilde{\boldsymbol{\Psi}}_\pi \boldsymbol{\Lambda}}_{\substack{\text{Propagation under}\\ \text{stickiness}}} \left[\underbrace{\boldsymbol{\alpha}}_{\substack{\text{Policy impact}\\ \text{via demand}}} \hat{Y}_t + \underbrace{\boldsymbol{\Omega}\boldsymbol{\mu}_t}_{\substack{\text{Impact of}\\ \text{cost-push shock}}} + \underbrace{\boldsymbol{\Omega}\boldsymbol{P}_{t-1}^P}_{\substack{\text{Impact of}\\ \text{lagged prices}}} + \underbrace{\beta \boldsymbol{\Lambda}^{-1} \mathbb{E}_t \boldsymbol{\pi}_{t+1}^P}_{\substack{\text{Forward-looking}\\ \text{behavior}}} \right]$$

Corollary 1. *The impact of a one-time tariff on the producer price inflation vector under price stickiness is:*

$$\frac{\partial \boldsymbol{\pi}_t^P}{\partial \tau_t} = \underbrace{\tilde{\boldsymbol{\Psi}}^{NKOE}}_{\substack{\textit{NKOE Leontief} \\ \textit{inverse}}} \quad \underbrace{\boldsymbol{\Lambda}}_{\textit{Stickiness}} \quad \underbrace{\tilde{\boldsymbol{\Omega}}^F}_{\substack{\textit{Tariff} \\ \textit{incidence}}}$$

where $\tilde{\boldsymbol{\Omega}}^F$ is a $NJ \times 1$ vector whose elements are the row sum of the foreign elements of $\boldsymbol{\Omega}$.

We can compare this with the impact under flexible prices:

$$\frac{\partial \boldsymbol{\pi}_t^{P,flex}}{\partial \tau_t} = \underbrace{(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}}_{\boldsymbol{\Psi}=\text{Leontief inverse}} \quad \underbrace{\tilde{\boldsymbol{\Omega}}^F}_{\text{Tariff incidence}} \tag{35}$$

Two points are noteworthy here. Firstly, since aggregate nominal demand—and consequently the exchange rate—is determined by policy, tariffs have no impact through the nominal exchange rate in this analytical model. However, the real exchange rate and the terms of trade do depend on tariffs. Secondly, the flexible-price expression captures a significant portion of the intuition. Under price stickiness, it is the propagation mechanism that changes, which is not surprising.

Proposition 2. *The impact of a one-time tariff ($\tau_t \geq 0$) on the producer price inflation is always weakly positive in the long run. That is let $\frac{\partial \boldsymbol{\pi}_t^P}{\partial \tau_t}$ be an $NJ \times 1$ vector, denoted as $\boldsymbol{\pi}_\tau^P$, such that $\boldsymbol{\pi}_\tau^P \geq \mathbf{0}$.*

Proof. Since the flexible-price equilibrium is the long run equilibrium, it would suffice to work with (35). We can express the matrix $(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$ as the following Neumann series:

$$(\boldsymbol{I} - \boldsymbol{\Omega})^{-1} = \sum_{k=0}^{\infty} \boldsymbol{\Omega}^k.$$

Each power $\boldsymbol{\Omega}^k$ has nonnegative entries, implying that $(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$ also has nonnegative entries. The term $\tilde{\boldsymbol{\Omega}}^F$ retains nonnegative entries. Since $(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$ is an $NJ \times NJ$ matrix with nonnegative entries and $\tilde{\boldsymbol{\Omega}}^F$ is an $NJ \times 1$ vector with nonnegative entries, their product is an $NJ \times 1$ vector with nonnegative entries. Thus, every entry of $\boldsymbol{\pi}_t^{P,flex}$ is weakly positive. $\square$

3.3.3 Consumer Price Inflation

With the NKPC describing producer price inflation, we next define consumer price inflation as follows. Aggregate consumption price indices in all countries are a linear combination of granular consumption prices, which in turn depend on producer prices, the exchange rate, and τ_t. Then, [9]

$$\hat{\boldsymbol{P}}_t^C = \boldsymbol{\Xi} \cdot \boldsymbol{P}_{n,t}^P + \underbrace{[\boldsymbol{\Xi} \odot \boldsymbol{\mathcal{E}}_t]\mathbf{1}}_{\tilde{\boldsymbol{L}}_{\mathcal{E}}^C \hat{\boldsymbol{M}}_t} + \underbrace{[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1}}_{\boldsymbol{L}_{\tau}^C \tilde{\boldsymbol{\tau}}_t} \tag{36}$$

where $\boldsymbol{\Xi}$ captures the share of each good i from country m in country n's consumption basket. Applying Lemma 1 from the Appendix, we can express

$$[\boldsymbol{\Xi} \odot \boldsymbol{\mathcal{E}}_t]\mathbf{1} = (\boldsymbol{I} - \boldsymbol{\Xi})\hat{\boldsymbol{M}}_t = \tilde{\boldsymbol{L}}_{\mathcal{E}}^C \hat{\boldsymbol{M}}_t.$$

Then, consumer price inflation can be written as:

$$\boldsymbol{\pi}_t^C = \Delta \hat{\boldsymbol{P}}_t^C = \boldsymbol{\Xi} \cdot \boldsymbol{\pi}_t^P + \tilde{\boldsymbol{L}}_{\mathcal{E}}^C \Delta \hat{\boldsymbol{M}}_t + \boldsymbol{L}_{\tau}^C \Delta \tilde{\boldsymbol{\tau}}_t \tag{37}$$

For simplicity, assuming lagged values are zero, i.e., $\hat{M}_{t-1} = \tau_{t-1} = 0$ (meaning the shock occurs at $t = 0$ and the economy was previously at steady state), and substituting the expression for producer price inflation from Proposition 1, we arrive at a solution for consumer price inflation. This solution maps lagged prices, policy, and tariffs to the consumer price inflation vector:

$$\begin{aligned}
\boldsymbol{\pi}_t^C = & \left(\boldsymbol{\Xi} \underbrace{\tilde{\boldsymbol{\Psi}}^{NKOE} \boldsymbol{\Lambda}}_{\substack{\text{NKPC} \\ \text{propagation}}} \quad \underbrace{(\boldsymbol{I} - \boldsymbol{\Omega})}_{\substack{\text{via demand and} \\ \text{via ER for producers}}} \quad + \quad \underbrace{(\boldsymbol{I} - \boldsymbol{\Xi})}_{\text{via ER for consumers}} \right) \hat{\boldsymbol{M}}_t \\
& + \left(\boldsymbol{\Xi} \underbrace{\tilde{\boldsymbol{\Psi}}^{NKOE} \boldsymbol{\Lambda}}_{\substack{\text{NKPC} \\ \text{propagation}}} \quad \underbrace{[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]}_{\substack{\text{Tariff incidence} \\ \text{for Producers}}} \quad + \quad \underbrace{[\boldsymbol{\Xi} \odot \hat{\boldsymbol{\tau}}_t]}_{\substack{\text{Tariff incidence} \\ \text{for consumers}}} \right) \mathbf{1} \\
& + \underbrace{\boldsymbol{\Xi} \left(\tilde{\boldsymbol{\Psi}}^{NKOE} - \boldsymbol{I} \right) \hat{\boldsymbol{P}}_{t-1}^P}_{\text{Impact of lagged prices}}
\end{aligned} \tag{38}$$

As seen above in Equation (38), policy and tariffs affect consumer price inflation through two channels: first, via producer prices, and second, through the exchange rate and tariffs that convert a producer price into a consumer price. A helpful interpretation of the expression above is that the terms labeled "NKPC Propagation" illustrate how the production network propagates shocks in a forward-looking setup, whereas the other terms represent the first-

[9] As described in Appendix C.1, we construct an $NJ \times NJ$ dimensional matrix $\boldsymbol{\Xi}$ and an $NJ \times 1$ dimensional consumer price vector by stacking each country's consumer demand matrix and consumer price vector.

order impacts. For example, when a τ_t% tariff is imposed, these terms capture what share of the consumption basket is affected, considering both its indirect effect through producers' input baskets and its direct effect on consumers' consumption baskets.

Proposition 3. *The impact of a one-time tariff ($\tau_t \geq 0$) on consumer price inflation is always weakly positive under fixed nominal demand. That is, let $\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t}$ be an $NJ \times 1$ vector such that $\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} \geq \mathbf{0}$.*

Proof. This follows from the logic of Proposition 2 and Equation (38). We can derive the necessary derivative from (38) as follows:

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \boldsymbol{L}_\tau^C + \boldsymbol{\Xi}\tilde{\boldsymbol{\Psi}}^{NKOE}\boldsymbol{\Lambda}\boldsymbol{L}_\tau^P \tag{39}$$

In this context, $\boldsymbol{L}_\tau^P = \tilde{\boldsymbol{\Omega}}^F$ and $\boldsymbol{L}_\tau^C = \tilde{\boldsymbol{\Xi}}^F$ correspond to the row sums of the foreign elements in intermediate inputs and final consumption, respectively. All matrices on the right-hand side of Equation (39) contain weakly positive entries. As a result, $\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} \geq \mathbf{0}$. □

Corollary 2. *Under flexible prices (efficient allocation), impact of tariffs on consumer prices consists of the following direct effects through the consumption basket and producer's input basket:*

$$\frac{\partial \boldsymbol{\pi}_t^{C^{flex}}}{\partial \tau_t} = \boldsymbol{L}_\tau^C + \boldsymbol{\Xi}\boldsymbol{\Psi}\boldsymbol{L}_\tau^P \tag{40}$$

and the difference between Equation (39) and Equation (40) yields the allocative efficiency term.

In this solution method with fixed nominal demand, the demand and exchange rate channels are fixed; tariffs have no impact on demand or the exchange rate, and these effects are linearly separable. In a setup like this, tariffs are always expected to be inflationary. To study the other two channels in a dynamic general equilibrium framework, we now turn to a different approach to solve the model.

3.4 Inflation Under a Taylor Rule: Two-Country Case

We now assume that $N = 2$, which allows us to solve for wages and the nominal exchange rate in the general expression in Equation (28) using a different approach, accounting for the endogenous impact of tariffs on demand and the exchange rate. To that end, we now assume that policy does not fix nominal domestic demand at an exogenously determined level. Instead, a Taylor rule is followed, given by $\hat{i}_t = \phi_\pi \pi_t^C$ as specified in the modeling framework.

As derived in Appendix D, forwarding the Euler equation yields the following approximate solution for consumption when shocks are transitory and ϕ_π is close to 1:

$$\hat{\boldsymbol{C}}_t = \overline{\boldsymbol{C}} - \frac{1}{\sigma}\boldsymbol{\Phi}(\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_{t-1}^C) \tag{41}$$

When the steady-state level of debt is made unique through portfolio adjustment costs, and as long as standard determinacy conditions are met (e.g., $\phi_\pi > 1$), it is guaranteed that $\lim_{t\to\infty} \hat{C}_t = \overline{C} = 0$, so Equation (41) serves as a solution that is numerically verified with the quantitative model. The intuition behind this expression becomes clearer by considering the limit $\phi_\pi \to 1$. In this case, we obtain $\hat{\boldsymbol{C}}_t = -\boldsymbol{\pi}_t^C$, which provides an exact solution indicating a downward-sloping aggregate demand curve once the Taylor Rule is substituted into the NKIS.

We additionally derive an approximate analytical solution for the nominal exchange rate. As shown in Appendix D, forwarding the UIP condition yields $\hat{\mathcal{E}}_t = \overline{E} + \phi_\pi \hat{P}_{t-1}^C - \phi_\pi^* \hat{P}_{t-1}^{*C}$ or in vector form, $\hat{\boldsymbol{\mathcal{E}}}_t = \overline{\boldsymbol{\mathcal{E}}}_t + \begin{bmatrix} 1 & -1 \end{bmatrix} \boldsymbol{\Phi} \hat{P}_{t-1}^C$.

From the structure of the UIP condition and the quantitative model, we know that in an exact analytical solution, the exchange rate is a function of contemporaneous shocks τ_t and lagged variables $\hat{\boldsymbol{P}}_{t-1}^C$ and $\hat{V}_{t-1}$. Our quantitative results indicate that when elasticities of substitution in production are below 1 (i.e., goods are complements), the balance of payments equation exhibits a high degree of persistence in real debt, even in the presence of portfolio adjustment costs. That is, when goods are difficult to substitute, the responsiveness of net exports to changes in the terms of trade—whether due to tariffs, exchange rates, or prices—diminishes. Consequently, when a shock occurs, it affects the balance of payments and the net debt position. However, the feedback from the lag of real debt $\hat{V}_t$ onto other variables is numerically small, particularly when cross-multiplied with other terms (e.g., as the lagged variable enters the NKPC equation through expectation terms). The contemporaneous impact of τ_t on the exchange rate arises via the equation of motion for debt. Once again, due to the low elasticity of substitution, this impact is numerically small as long as shocks are transitory.[10] Moreover, these terms are linearly separable. For this reason, in the analytical solution, we assume $\overline{E} \approx 0$ and use the following expression to substitute out the

[10] Our approach here and simplifications are only valid to study transitory shocks. As we show with our quantitative results, in the presence of near-permanent tariff shocks the exchange rate will jump when tariffs are announced.

exchange rate from the equilibrium conditions:

$$\hat{\boldsymbol{\mathcal{E}}}_t = \begin{bmatrix}1 & -1\end{bmatrix} \boldsymbol{\Phi} \hat{P}^C_{t-1} \tag{42}$$

Setting labor elasticity to $\gamma = 0$, as we did earlier in this section, the labor-leisure condition once again gives:

$$\hat{\boldsymbol{W}}_t = \hat{\mathbf{P}}^C_t + \sigma \hat{\boldsymbol{C}}_t = (\mathbf{I} - \boldsymbol{\Phi})\hat{\mathbf{P}}^C_t - \boldsymbol{\Phi}\boldsymbol{P}^C_{t-1}$$

Plugging these into Equation (28), grouping terms, and rearranging, we obtain:

$$\hat{\boldsymbol{P}}^P_t = \tilde{\boldsymbol{\Psi}}_{\boldsymbol{\phi}} \left[\hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{\Lambda}\left((\boldsymbol{L}^P_C + \mathbf{L}^P_{\hat{\mathcal{E}}})\boldsymbol{\Phi}\boldsymbol{P}^C_{t-1} + \left[\boldsymbol{L}^P_C(\mathbf{I} - \boldsymbol{\Phi})\boldsymbol{L}^C_\tau + \mathbf{L}^P_\tau\right]\tau_t \right) + \beta \mathbb{E}t \hat{\boldsymbol{P}}^P_{t+1} \right] \tag{43}$$

where $\tilde{\boldsymbol{\Psi}}_\phi = \left[\boldsymbol{I}(1+\beta) - \boldsymbol{\Lambda}\left[\boldsymbol{\Omega} - \boldsymbol{I} + \boldsymbol{L}^P_C(\boldsymbol{I} - \boldsymbol{\Phi})\boldsymbol{\Xi}\right] \right]^{-1}$ is now the *stickiness and policy-adjusted Leontief Inverse*.

Using (41) and (42) we can substitute out $\hat{\boldsymbol{C}}_t$ and $\hat{\boldsymbol{\mathcal{E}}}_t$ in the CPI equation in (27) and the equation of motion for debt in (30). Combining the resulting expressions with (43) we have a block that maps τ_t to $\hat{\mathbf{P}}^P_t, \hat{\mathbf{P}}^C_t, \hat{V}_t$. With that, once again using the method of undetermined coefficients, we can find an analytical solution. We confirm that our solution is numerically accurate, especially when ϕ_π is close to 1.[11] Additionally, in Appendix D.5.1 we show how our solution can collapse to the standard solution of the three-equation New Keynesian model when $N = 1$ and $J = 1$.

Proposition 4. *The impact of a one-time tariff on CPI inflation is*

$$\frac{\partial \boldsymbol{\pi}^C_t}{\partial \tau_t} = \boldsymbol{\Xi}\tilde{\boldsymbol{\Psi}}^{NKOE}_\phi \boldsymbol{\Lambda}\left[\boldsymbol{L}^P_\tau + \left(\boldsymbol{L}^P_C(\mathbf{I} - \boldsymbol{\Phi}) + \beta(\boldsymbol{L}^P_C + \boldsymbol{L}^P_{\mathcal{E}})\boldsymbol{\Phi}\boldsymbol{L}^C_{\mathcal{E}} \right)\boldsymbol{L}^C_\tau \right] + \boldsymbol{L}^C_\tau \tag{44}$$

where $\boldsymbol{L}^C_{\mathcal{E}} = \overline{\rho}(\boldsymbol{I} - \beta\overline{\rho}\tilde{\boldsymbol{L}}^C_{\mathcal{E}})^{-1}$, *and* $\tilde{\boldsymbol{\Psi}}^{NKOE}_\phi$ *is the stickiness- and policy-adjusted NKOE Leontief inverse. This expression endogenizes the demand and exchange rate response to the imposition of tariffs.*[12] *It transforms the stickiness- and policy-adjusted Leontief inverse* $\tilde{\boldsymbol{\Psi}}_\phi$ *by diagonalizing it and solving a quadratic equation to determine the matrix in front of the diagonalized lagged vector* $\breve{\boldsymbol{P}}^P_{t-1}$, *denoted as* $\mathbf{C}_1$. *The NKOE Leontief inverse* $\tilde{\boldsymbol{\Psi}}^{NKOE}_\phi$ *is then*

[11] In our baseline comparison with both countries' parameter set to $\phi_\pi = \phi^*_\pi = 1.01$, our Dynare simulation finds US inflation to be 0.8123%, while our linearized approximation matrices find this impact to be 0.8104%.

[12] The dimensions of the loadings are as follows: $\boldsymbol{L}^P_\tau$ is $NJ \times 1$, $\boldsymbol{L}^P_C$ is $NJ \times N$, $\boldsymbol{L}^P_{\mathcal{E}}$ is $NJ \times N$, $\boldsymbol{L}^C_{\mathcal{E}}$ is $N \times N$, $\boldsymbol{L}^C_\tau$ is $N \times 1$.

constructed by taking this solution $\boldsymbol{C}_1$ *and applying the transformation:*$\tilde{\boldsymbol{\Psi}}_{\phi}^{NKOE} = \boldsymbol{Q}\boldsymbol{C}_1\boldsymbol{Q}^{-1}$*, where* $\boldsymbol{Q}$ *diagonalizes* $\tilde{\boldsymbol{\Psi}}_{\phi}^{NKOE}$*. In this formulation,* $\tilde{\boldsymbol{\Psi}}_{\phi}^{NKOE}$ *also serves as the matrix multiplying the lagged price vector when producer price inflation is expressed in levels. The matrix* $\mathbf{Q}$ *represents the collection of eigenvectors that diagonalize the stickiness-adjusted Leontief inverse,* $\tilde{\boldsymbol{\Psi}}$*.*

Proof. See Appendix D. □

By once again annotating direct effects in orange and indirect effects in purple, we can highlight key insights regarding the five indirect reallocation channels that extend beyond the direct impact of tariffs on CPI and PPI: (i) the contemporaneous demand channel inclusive of policy, (ii) the expected demand channel inclusive of policy, (iii) the expected exchange rate channel, (iv) price stickiness, and (v) the network channel.

Equation (44) indicates that, beyond direct effects, tariffs also influence inflation through contemporaneous and expected demand, as well as through the exchange rate. These effects are propagated via the NKOE Leontief inverse and price stickiness. Expectations, captured by the scalar β, affect inflation both through the term $\left(\boldsymbol{L}_C^P + \boldsymbol{L}_{\mathcal{E}}^P\right)\boldsymbol{\Phi}\boldsymbol{L}_{\mathcal{E}}^C\boldsymbol{L}_{\tau}^C$ and through the NKOE Leontief inverse, which incorporates β. Consequently, when tariffs are imposed, they generate direct inflationary pressure. However, this effect is partially mitigated by a decline in demand and reduced international risk sharing. Given the presence of lags in the system, these indirect effects are transmitted through the expected inflation channel. The contemporaneous demand channel operates under the assumption that aggregate demand slopes downward in prices – in the New Keynesian framework, this arises because the central bank raises real interest rates in response to increased consumption, thereby contracting demand.

The core intuition behind the exchange rate channel can be explained as follows. Abstracting from the stationarity-inducing device of portfolio adjustment costs, forwarding the UIP condition yields:

$$\hat{\mathcal{E}}_t = \overline{\mathcal{E}} + \ E_t\left[\sum_{j=0}^{\infty}(\phi_\pi^*\pi_{t+j}^* - \phi_\pi\pi_{t+j})\right]$$

where $\overline{\mathcal{E}} = \lim_{t\to\infty}\hat{\mathcal{E}}_t$. Based on our analytical and quantitative findings, and in the context of temporary tariffs, we can momentarily assume that $\overline{\mathcal{E}}$ is numerically small and linearly separable from current and expected inflation terms. Given the differing weights that central banks assign to inflation, the country with the higher expected inflation path multiplied with the weight on inflation will experience a nominal currency appreciation. This follows the intuition of the Backus-Smith condition and the principle of international risk sharing as well: when the consumption basket in one country becomes more expensive, capital flows toward that country to equalize marginal utilities across borders. These inflows raise the value of that country's currency, making foreign goods relatively cheaper and thus more attractive. This dynamic is stronger when the central bank of the country with the higher inflation also places higher weight on inflation.

In line with the discussion on the exchange rate offset, it is important to analyze how the exchange rate loads onto consumer prices: $\boldsymbol{L}^C_{\mathcal{E}} = \overline{\rho}(\boldsymbol{I} - \beta\overline{\rho}\tilde{\boldsymbol{L}}^C_{\mathcal{E}})^{-1}$. The matrix $\tilde{\boldsymbol{L}}^C_{\mathcal{E}}$ captures the weighted composition of goods affected by the exchange rate. Specifically, it reflects home bias parameters for the two countries (e.g., $\gamma_b, \gamma_b^* \in [0, 1]$), each subtracted from 1. The term $\boldsymbol{L}^C_{\mathcal{E}}$ thus represents a geometric series summing the repeated effects of these weights over time:

$$\tilde{\boldsymbol{L}}^C_{\mathcal{E}} = \begin{bmatrix} 1-\gamma_b \\ 1-\gamma_b^* \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} = \begin{bmatrix} 1-\gamma_b & \gamma_b - 1 \\ 1-\gamma_b^* & \gamma_b^* - 1 \end{bmatrix} \quad \rightarrow \quad \boldsymbol{L}^C_{\mathcal{E}} = \sum_{k=0}^{\infty} \beta^k (\tilde{\boldsymbol{L}}^C_{\mathcal{E}})^k$$

Further intuition can be gained by comparing the solution in Equation (39) under fixed nominal demand to that in Equation (44) under a Taylor rule. In the former, the impact on demand and the exchange rate is linearly separable from tariffs. Thus, the two expressions differ in the following ways: (i) in how the NKOE Leontief inverse is reshaped by policy, and (ii) through the term $\left(\boldsymbol{L}^P_C(\mathbf{I} - \boldsymbol{\Phi}) + \beta(\boldsymbol{L}^P_C + \boldsymbol{L}^P_{\mathcal{E}})\boldsymbol{\Phi}\boldsymbol{L}^C_{\mathcal{E}}\right)\boldsymbol{L}^C_{\tau}$. This term captures how tariffs impact contemporaneous demand, expected demand, and expected exchange rates. Part of this impact operates through lagged consumer prices, which enter contemporaneous inflation via the expected inflation term in the Phillips Curve, hence the presence of β in the expression. We can analyze this term further by separating it into its three components:

$$\underbrace{\boldsymbol{L}^P_C(\mathbf{I} - \boldsymbol{\Phi})}_{\substack{\text{Tariff Impact} \\ \text{via Demand}}} + \beta \underbrace{\boldsymbol{L}^P_C\boldsymbol{\Phi}\boldsymbol{L}^C_{\mathcal{E}}\boldsymbol{L}^C_{\tau}}_{\substack{\text{Tariff Impact} \\ \text{via Expected Demand}}} + \beta \underbrace{\boldsymbol{L}^P_{\mathcal{E}}\boldsymbol{\Phi}\boldsymbol{L}^C_{\mathcal{E}}\boldsymbol{L}^C_{\tau}}_{\substack{\text{Tariff Impact} \\ \text{via Expected ER}}}$$

The way a term loads onto $\hat{C}_t$ and $\hat{\mathcal{E}}_t$ is by first loading onto consumer prices. In this sense, $\boldsymbol{L}^P_{\mathcal{E}}\boldsymbol{\Phi}\boldsymbol{L}^C_{\mathcal{E}}\boldsymbol{L}^C_{\tau}$ captures how tariffs affect consumer prices, which in turn impact the exchange rate, thereby influencing producer prices. Similarly, $\boldsymbol{L}^P_C\boldsymbol{\Phi}\boldsymbol{L}^C_{\mathcal{E}}\boldsymbol{L}^C_{\tau}$ captures how tariffs load onto consumer prices and, consequently, influence demand. As this process unfolds, these effects are mediated by policy, as captured by $\boldsymbol{\Phi}$.

Remark 1. The sign of inflation depends, in part, on how the exchange rate loads onto PPI and CPI, as captured by $\boldsymbol{L}^P_{\mathcal{E}}$ and $\boldsymbol{L}^C_{\mathcal{E}}$. Since these terms contain negative entries, their overall influence can vary. The network structure, when combined with price stickiness and sectoral heterogeneity, can either amplify or dampen some loadings, thereby shaping the overall sign and magnitude of inflation.

In discussing when and why the input–output structure matters, it is important to note that the network propagation in Equation (44)—captured by $\tilde{\mathbf{\Psi}}_\phi^{NKOE}$ and amplified or muted by $\mathbf{\Lambda}$—can overturn some expected signs (e.g., tariffs being inflationary), depending on which entries are emphasized. For instance, if the term $\tilde{\mathbf{\Psi}}_\phi^{NKOE}\mathbf{\Lambda}$ amplifies negative components—due to rigidities and how they load—then tariffs could, in principle, have a deflationary effect.

The analytical solution allows us to decompose the impact of tariffs into distinct channels, which we map to the mechanisms discussed above. These channels correspond directly to the input–output matrix, consumption shares, and deep structural parameters. As such, they can serve as model-based, ex-ante sufficient statistics.[13]

$$\begin{aligned}\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = & \underbrace{\Xi \boldsymbol{L}_\tau^P}_{\text{Direct PPI effect}} + \underbrace{\Xi \boldsymbol{L}_C^P(\mathbf{I}-\mathbf{\Phi})\boldsymbol{L}_\tau^C}_{\text{Demand channel}} + \underbrace{\beta \Xi \boldsymbol{L}_C^P \mathbf{\Phi} \boldsymbol{L}_\mathcal{E}^C \boldsymbol{L}_\tau^C}_{\text{Expected demand channel}} + \underbrace{\beta \Xi \boldsymbol{L}_\mathcal{E}^P \mathbf{\Phi} \boldsymbol{L}_\mathcal{E}^C \boldsymbol{L}_\tau^C}_{\text{Expected ER channel}} + \underbrace{\boldsymbol{L}_\tau^C}_{\text{Direct CPI effect}} \\ & + \underbrace{\Xi(\tilde{\mathbf{\Psi}}_\phi^{NKOE}\mathbf{\Lambda} - \mathbf{I})\mathbf{Z}}_{\text{Propagation}} \end{aligned} \tag{45}$$

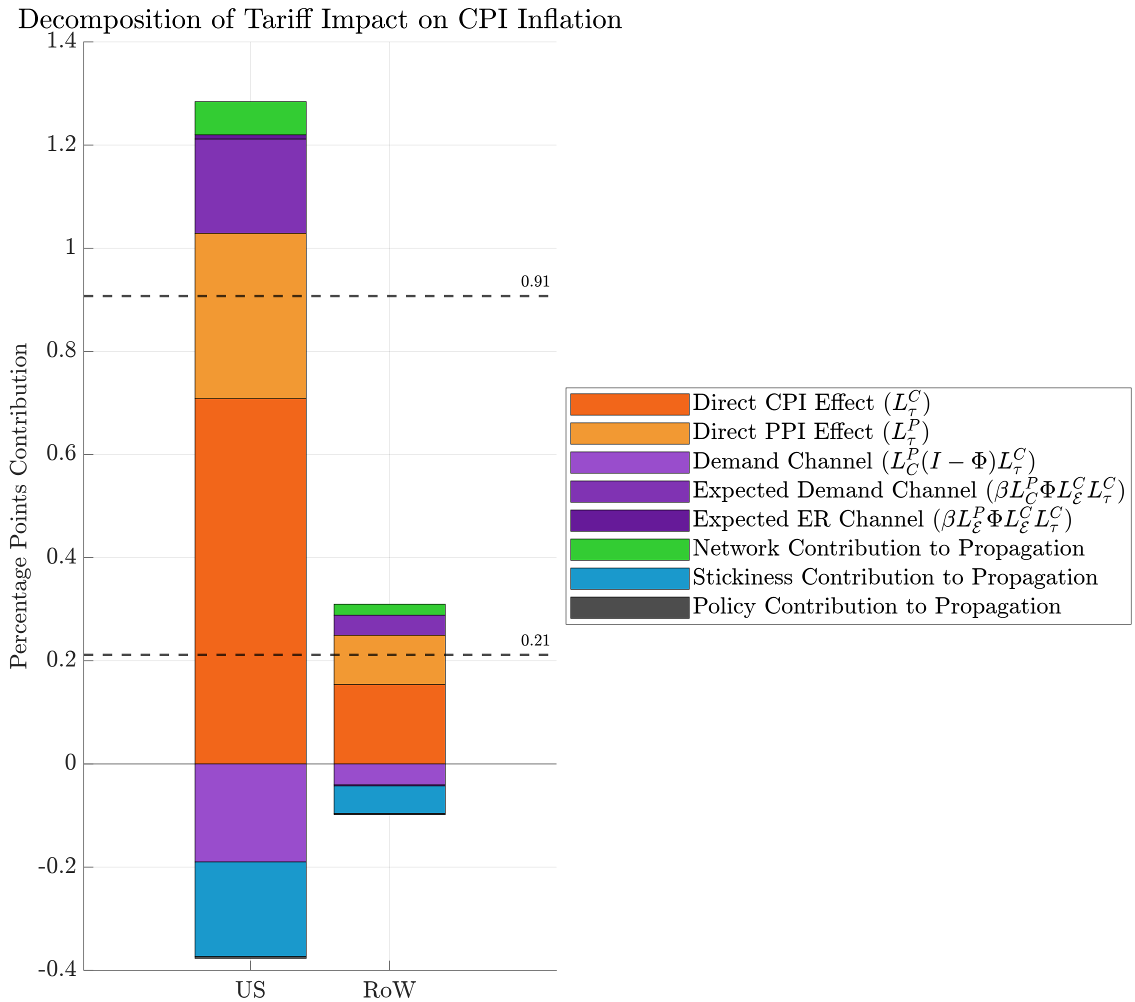
The propagation term captures the combined impact of the input–output structure, price stickiness, and policy. These components are difficult to analytically disentangle due to the definition of the stickiness- and policy-adjusted Leontief inverse prior to solving in the NKOE setting: $\tilde{\mathbf{\Psi}}_\phi = \left[\boldsymbol{I}(1+\beta) - \mathbf{\Lambda}\big[\mathbf{\Omega} - \boldsymbol{I} + \boldsymbol{L}_C^P(\boldsymbol{I}-\mathbf{\Phi})\Xi\big]\right]^{-1}$. For this reason, we numerically decompose the propagation term into the contributions of $\mathbf{\Omega}$, $\mathbf{\Phi}$, and the remainder. Specifically, we set $\mathbf{\Omega} = \mathbf{0}$ and $\mathbf{\Phi} = \mathbf{I}$ one at a time, labeling these as the contributions of the network and policy to propagation, respectively. The remaining portion is attributed to price stickiness.

To illustrate how these channels operate and to build intuition around the model, let us consider an example based purely on the analytical solution above. Our objective here is not to conduct a full quantitative exercise—that is reserved for Section 4. Imagine dividing the world into two regions: the United States and the rest of the world. Suppose the United States imposes a 10% tariff on all goods and industries imported from the rest of the world for one period. In response, the rest of the world retaliates during the same period. Agents in both regions anticipate that these tariffs are transitory and will be lifted in the following period. We use the parameter values described in greater detail in Section 4 and Table 1, except where simplifications of the analytical model apply (e.g., $\sigma = 1$, $\gamma = 0$). The impact

[13] Details are available in Appendix F.

of this theoretical tariff shock is illustrated in Figure 2 below.

Figure 2. US Against the Rest: Decomposing Impact of Global Tariff War



Note: Here, we decompose CPI inflation in a two-country case, namely the U.S. and the rest of the world (RoW). We assume both regions impose an additional 10% tariff on each other. Using Equation 45, we break down the different contributing effects. The dashed line represents the total effect, showing an inflation increase of 0.91% in the U.S. and 0.21% in the rest of the world. In this theoretical example based on our analytical solution, we use annual price updating frequencies, whereas in the quantitative model we use quarterly frequencies.

When this transitory tariff shock occurs, the direct impact on CPI and PPI generates an inflationary impulse of approximately 1 percentage point in the tariff-imposing country. The magnitude of these direct affects is related to the trade openness of the United States. Beyond these direct effects, we also observe indirect effects. As expected, the contemporaneous demand channel carries a negative sign. Under policy, aggregate demand slopes downward in response to inflation. In the New Keynesian framework, this arises because the central bank raises real interest rates in response to rising consumption, thereby contracting demand. Consequently, when the tariff shock hits, agents choose to forego consumption today in favor of consuming tomorrow. Meanwhile, the expected demand channel generates an additional inflationary impulse. The contemporaneous demand channel, however, exerts downward pressure on inflation, as agents anticipate that the tariffs are one-time, transitory shocks and expect them to dissipate in the following period.

The exchange rate response is relatively muted due to home bias, an elasticity of substitution below one, and the transitory nature of the shock. What partially offsets the initial inflationary impulse of approximately 1.3 percentage points—bringing the overall effect down to 0.91 percentage points—is the combined influence of price stickiness and the contemporaneous demand channel. The primary impact of policy operates through contemporaneous demand; policy's overall contribution to propagation is limited. In contrast, the input–output network generates positive inflationary pressure—a mechanism we explore in greater detail in Section 3.5.

Note that, first, the impact on the rest of the world follows the same directional pattern as in the United States but is smaller in magnitude. This is because the rest of the world is larger than the U.S., making the distortion a relatively smaller shock in the context of the global economy. Second, as discussed in Section 4, the contemporaneous exchange rate response—abstracted from in this section—exhibits larger movements when the shock is permanent. When the shock is transitory and elasticities of substitution are below one, meaning goods behave as complements in production, the exchange rate response is muted. As indicated by the analytical expression used in this section, the exchange rate closely follows changes in the price level. Third, this figure underscores what the full model captures compared to standard trade or small open economy (SOE) models. In the absence of intertemporal optimization and forward-looking behavior, the contemporaneous demand channel—as well as the expected demand and expected exchange rate channels—would be absent. In the SOE case, loadings from the rest of the world would not be present. Finally, in models without network effects, the network channel would also be absent.

These channels have intuitive interpretations and relate to different economic scenarios. Consider, for example, a case in which the production basket has very limited exposure to the rest of the world, while final goods consumption is highly open to international trade. In this limiting case, Equation (44) simplifies to:

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \boldsymbol{\Xi} \tilde{\boldsymbol{\Psi}}_{\phi}^{NKOE} \boldsymbol{\Lambda} \left[\left(\boldsymbol{L}_C^P (\mathbf{I} - \boldsymbol{\Phi}) + \beta \boldsymbol{L}_C^P \boldsymbol{\Phi} \boldsymbol{L}_{\mathcal{E}}^C \right) \boldsymbol{L}_{\tau}^C \right] + \boldsymbol{L}_{\tau}^C.$$

In this scenario, the demand shock component of tariffs can more easily outweigh the supply shock component, leading to lower inflation and lower output. We can observe the impact of this case by removing the Direct PPI Effect in Figure F and adjusting for propagation. At the opposite extreme, consider a case where final goods consumption is relatively closed to international trade, while the production side has significant exposure to foreign inputs. In this case, Equation (44) simplifies to: $\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \boldsymbol{\Xi}\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda}\boldsymbol{L}_\tau^P$, which implies that the supply shock component dominates the demand component. With the appropriate adjustment for propagation, this scenario can also be mapped to the decomposition shown in Figure F.

3.5 Impact of the $\boldsymbol{\Omega}$, $\boldsymbol{\Lambda}$, $\boldsymbol{\Phi}$ and $\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}$ Matrices

Given that price stickiness and input–output linkages are both central components of the framework, we now turn to the question of how the network channel operates, how the structure of the $\boldsymbol{\Omega}$ matrix influences transmission and how this interacts with price stickiness. Why is the sign of the network channel in Figure F positive? To understand this and the broader impact of the network channel we begin by establishing several key insights that follow from the propositions above.[14]

Remark 2. In a first-order approximation setting, the regular Leontief inverse ($\boldsymbol{\Psi} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$) and the stickiness-adjusted Leontief inverse, multiplied by the stickiness matrix ($\tilde{\boldsymbol{\Psi}}_\pi\boldsymbol{\Lambda} = (\boldsymbol{I} - \boldsymbol{\Lambda}(\boldsymbol{\Omega} - \boldsymbol{I}))^{-1}\boldsymbol{\Lambda}$), will behave similarly, provided there is no heterogeneity in the stickiness parameter across sectors. The key difference between them lies in the magnitude of inflation.

Proof. Each of the two objects involve Neumann series. In the absence of sectoral heterogeneity, $\boldsymbol{\Lambda} = \lambda\boldsymbol{I}$. Then:

$$\boldsymbol{\Psi} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} = \sum_{k=0}^{\infty} \boldsymbol{\Omega}^k$$

Similarly, for the stickiness-adjusted Leontief inverse:

$$\begin{aligned}\tilde{\boldsymbol{\Psi}}_\pi\boldsymbol{\Lambda} &= (\boldsymbol{I} - \boldsymbol{\Lambda}(\boldsymbol{\Omega} - \boldsymbol{I}))^{-1}\boldsymbol{\Lambda} \\ &= \frac{\lambda}{1+\lambda}\sum_{k=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^k \boldsymbol{\Omega}^k.\end{aligned}$$

As long as $\boldsymbol{\Omega}_{ij} \neq 0$ for some i, j, the relative importance—or centrality—of sectors remains unchanged in the absence of heterogeneity in price stickiness across sectors. However, the

[14] Here, we focus on our linearized model. More broadly, the structure of $\boldsymbol{\Omega}$ is important because, at higher orders, a more detailed depiction of the production network significantly affects outcomes, especially when shocks are large. To a first-order approximation, however, the aggregation of any CES bundle behaves similarly to a Cobb–Douglas structure.

overall impact on inflation will be scaled by a constant factor. As established in Rubbo (2023), when a finer I-O matrix captures more goods within the $\mathbf{\Omega}$ matrix, the aggregate Phillips Curve becomes flatter. This occurs because, as the number of sectors increases, the individual input–output coefficients $\mathbf{\Omega}_{ij}$ decrease, reflecting a more granular production network. Since $\mathbf{\Omega}$ enters the Neumann series multiplicatively, and assuming $\mathbf{\Omega}_{ij} \in (0, 1)$, smaller $\mathbf{\Omega}_{ij}$ entries attenuate the aggregate impact of sectoral shocks. As a result, the aggregate Phillips Curve flattens: nominal rigidities become more diffuse across a fragmented network, reducing the responsiveness of inflation to shocks. Consequently, as prices respond less, quantities respond more. □

Remark 3. In a NKOE setting, as shown in Equation (44), the combination of cross-sectoral heterogeneity in the price stickiness term $\mathbf{\Lambda}$ and the stickiness- and policy-adjusted NKOE Leontief inverse, $\tilde{\mathbf{\Psi}}^{NKOE}$, can exert downward pressure on inflation. This occurs in part because $\tilde{\mathbf{\Psi}}^{NKOE}$ is not restricted to having weakly positive entries. When there is heterogeneity in price stickiness or policy preferences—either across sectors or across countries—$\tilde{\mathbf{\Psi}}^{NKOE}$ can amplify negative entries from other channels, further dampening the aggregate inflation response.

In our context, $\mathbf{\Lambda}$ captures price stickiness; an extreme case of flexible prices corresponds to a vertical supply curve with a fixed quantity. In this scenario, the inelasticity of supply can amplify the influence of a given sector or country. Suppose a particular country–sector combination constitutes only a small share of the home country's producer price basket. If its supply is inelastic, the NKOE impact of a tariff on this country–sector will be disproportionately large. This type of effect may be overlooked in models where all intermediate goods are bundled together under flexible pricing.

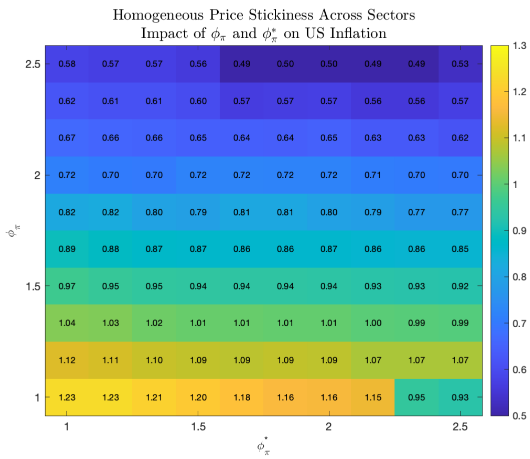
Relatedly, omitting a fully specified input–output structure can lead to an overestimation of inflation and an underestimation of the quantity response, including effects on unemployment. Suppose intermediate inputs are bundled with some degree of price stickiness (or even under flexible pricing), while, for notational simplicity, final goods production follows either flexible or staggered pricing. In such a case, inflation in final goods may be overstated, since the network channel can exert downward pressure on overall inflation.

Remark 4. Given interlinkages between sectors, heterogeneity in price stickiness parameters can compress the range of inflation outcomes that the central bank can achieve through endogenous rate hikes under a Taylor Rule, thereby reducing the effectiveness of monetary policy.

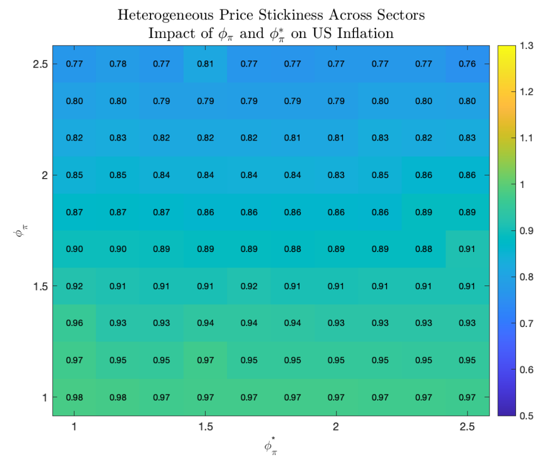
Rubbo (2023) notes that eliminating heterogeneity in stickiness parameters does not necessarily alter the slope of the aggregate Phillips Curve. This outcome depends on how the average price updating frequency is computed, how correlated labor, input and consumption shares are. While we do not report Phillips Curve slopes in our setting, the exercise we have in mind is similar in spirit.

Figure 3. Impact of Heterogeneity: Price Stickiness vs. ϕ_π

(a)



(b)



Note: Heatmaps show U.S. CPI inflation in a two-country setting (the United States and the rest of the world), where both regions impose a 10% tariff on each other. The horizontal and vertical axes vary the inflation response parameters ϕ_π and ϕ^*_π in the Taylor Rule for the U.S. and the rest of the world, respectively. The heatmaps reflect the resulting U.S. inflation as these policy parameters vary.

With the model analytically solved, what can we say about the NKOE impact on inflation from changing the weight on inflation in the Taylor Rule, ϕ_π? And how is this impact affected by heterogeneity in price updating frequencies? We explore these questions in Figure 3. These heatmaps are based on the analytical solution and reflect the same setup as in Figure 2, where the United States imposes 10% tariffs on the rest of the world, and the rest of the world retaliates.[15] The two axes in each heatmap vary the central banks' weights on inflation, ϕ_π and ϕ^*_π, in the two blocs. The heatmap color indicates the resulting inflation in the United States. The right-hand panel shows the case in which price stickiness parameters are heterogeneous across sectors, using the values from our full quantitative simulations based on Nakamura and Steinsson (2008).[16] The left-hand panel shows the case in which a single stickiness parameter is applied to all sectors. To match the overall magnitude across

[15] Since our analytical solution involves approximations, we have verified the relative magnitudes and ranges of these estimates using `Dynare`.

[16] We conducted simulations using alternative stickiness parameterizations, including Monte Carlo simulations with randomly drawn vectors of sectoral stickiness. Across these exercises, we consistently find that heterogeneity in stickiness compresses the range of inflation outcomes attainable by varying ϕ_π.

both panels, the stickiness parameter used in Figure 3a is set equal to the sales-weighted average of sectoral stickiness.[17] These figures suggest that, in our context and using the ICIO input–output table, cross-sectoral heterogeneity in price stickiness compresses the range of inflation outcomes that the central bank can achieve through endogenous rate hikes: from 0.49% to 1.23% in the homogeneous case, versus 0.76% to 0.98% in the heterogeneous case.

Two additional observations are worth noting. First, this result is specific to the input–output (I–O) table we use. Intuitively, and based on Equation (44), the slope of the Phillips Curve matters for how $\mathbf{\Phi}$ affects inflation, and this, in turn, depends on $\boldsymbol{L}_C^P$—which, in the context of the analytical solution, contains only labor shares.[18] Then it will matter if sectors with high vs. low labor shares get higher or lower stickiness parameters as Rubbo (2023) notes. Second, the simulations in Figure 3 suggest that, when tariffs are modeled as one-time transitory shocks, heterogeneity in ϕ_π across countries does not significantly affect inflation outcomes in the home country. However, in quantitative simulations using a multi-country setup, we find that the response of variables such as the exchange rate and inflation to near-permanent shocks does depend on cross-country heterogeneity in ϕ_π.

Remark 5. The matrix of price stickiness parameters $\mathbf{\Lambda}$ influences inflation in three different ways: (i) via the average level of price stickiness,[19] (ii) via cross-sectoral heterogeneity, whereby it will matter if a sector with high vs. low labor shares get higher or lower stickiness parameters, and (iii) via the interaction with $\mathbf{\Omega}$ inside the NKOE Leontief inverse.

Of the three ways in which $\mathbf{\Lambda}$ influences inflation, the first is present both in models without multiple sectors and without input-output linkages. The second could be present in a model that has multiple sectors without explicitly introducing input-output linkages. However, the third can only be present in models with input-output linkages.

This brings up our final point regarding the impact of $\mathbf{\Omega}$ on inflation; this impact is a nuanced one. On the one hand, having a finer or more granular network flattens the Phillips Curve and as such would mute the impact of shocks on inflation as outlined above. On the other hand, the very reliance of one sector on another introduces positive weights inside the marginal cost expression for each sector such that for a given network $\mathbf{\Omega}$ will have a positive impact on inflation. This second and positive impact is what makes the network contribution to propagation positive in Figure 2. Inside the stickiness and policy-adjusted Leontief Inverse, $\mathbf{\Omega}$ is multiplied by $\mathbf{\Lambda}$ before we arrive at the NKOE Leontief Inverse.

[17]Specifically, we take the weighted average of the diagonal entries of $\mathbf{\Lambda}$.

[18]This is because labor is assumed to be elastic in the analytical solution, so there is no Frisch elasticity term in the slope of the PPI New Keynesian Phillips Curve.

[19]As noted in Rubbo (2023) how this average is calculated matters. Averaging Calvo price updating frequencies first and then calculates a single price stickiness parameter yields a different result than calculating price stickiness parameters and then averaging them. We find it also matters whether the final scalar price stickiness parameter that is used is a weighted average or a simple average.

This implies that the positive inflationary impulse from input-output linkages are highly dependent on the distribution of price stickiness parameters. If a given sector's reliance on an input from another sector is multiplied by a high (low) price stickiness parameter, the inflation (quantity) impact from a shock to that sector will be amplified. Put differently, using different price stickiness parameters can make the network contribution to propagation larger in Figure 2.

Intuitively, if a given sector is central to production whether because it is widely used in different industries (e.g., steel and aluminum) or its downstream linkages (e.g., semiconductor chips)—it will carry a high weight in the standard Leontief inverse. If this sector also exhibits highly flexible (rigid) prices indicating a vertical (horizontal) supply curve with fixed quantity (highly elastic supply),, the inflationary impact of a tariff on this sector will be amplified (muted) by $\tilde{\mathbf{\Psi}}_{\phi}^{NKOE}$. Since $\tilde{\mathbf{\Psi}}_{\phi}^{NKOE}$ also includes distribution of central banks' weights on inflation, whether the shocks hit countries with loose or tight monetary policy will be an additional amplification or deamplification channel.

4 Quantitative Results

4.1 Calibration

We now return to the non-linear, exact version of the model without any simplifying assumptions. Tariffs follow an AR(1) process (i.e., $\tau_t = \rho^{\tau}\tau_{t-1} + \epsilon_t^{\tau}$) and we specify the value of ρ^{τ} assumed in each example. The quantitative model also incorporates a permanent real capital account wedge to treat the year 2018 as the steady state to which the economy eventually returns. This allows us to embed a realistic net foreign asset (NFA) position for all relevant country blocks. Without these wedges, calibrating the steady state to match a given year's data would imply, for instance, that a country running a current account deficit in steady state must also be a net lender, with its net imports financed by returns on net foreign assets—an unrealistic outcome. The introduction of capital account wedges addresses this issue while preserving empirical consistency.

For calibration, we follow the approaches of Di Giovanni et al. (2023) and Itskhoki and Mukhin (2021). The calibration parameters are summarized in Table 1. The model employs sector-specific Calvo parameters based on the empirical estimates in Nakamura and Steinsson (2008), adjusted to a quarterly frequency. The production and intratemporal consumption structures are similar to those in Çakmaklı et al. (Conditionally accepted) and Di Giovanni et al. (2023). On the production side, firms combine labor and intermediate input bundles to produce goods. Based on Atalay (2017), we set the elasticity of substitution between labor and intermediates to $\theta = 0.6$.[20] Intermediate input bundles are composed of sectoral bundles, which are assumed to be complements. Following Boehm et al. (2019), we set the elasticity for this aggregation stage to $\theta_h = 0.2$. Each sectoral bundle consists of varieties sourced from different countries. In our baseline specification, we set the Armington elasticity across countries at the sectoral level to $\theta_l^i = 0.6$. On the intratemporal consumption side, we follow Baqaee and Farhi (2024) and assume Cobb–Douglas preferences across sectors, setting the sectoral elasticity to $\sigma_h = 1$. For the aggregation of varieties within sectoral consumption bundles, we adopt the same approach as in the production structure.

Additionally, in the quantitative model, we incorporate monetary policy inertia by modifying the baseline Taylor rule. Specifically, Equation (24) is replaced with the following specification:

$$1 + i_{n,t} = (1 + i_{n,t-1})^{\rho_m^n} (\Pi_{n,t})^{\phi_\pi^n} (Y_{n,t})^{\phi_y^n} e^{\hat{M}_{n,t}} \quad \forall n \in N$$

Here, ρ_m^n captures the degree of interest rate smoothing (or policy inertia), ϕ_π^n and ϕ_y^n are the inflation and output coefficients in the Taylor rule, and $\hat{M}_{n,t}$ denotes a monetary policy shock. This specification is applied to all countries $n \in N$ in the model.

For the United States, we set $\rho_m^{\mathrm{US}} = 0.82$ and $\phi_\pi^{\mathrm{US}} = 1.29$, based on the estimates provided by Carvalho et al. (2021). Following Clarida et al. (2000), we use $\rho_m^n = 0.95$ and $\phi_\pi^{EA} = 1$ for the rest of the world and the euro area, respectively.For other countries in the rest of the world, we assume $\phi_\pi^n = 0.2$, except for Mexico, where we use a slightly higher value of $\phi_\pi^{\mathrm{MX}} = 0.3$. These ϕ_π values are calibrated using a model-consistent interpretation of the long-run average of quarterly inflation rates. Specifically, following the logic in Clarida et al. (2000), we set $\phi_\pi^n = \frac{1-\rho_m^n}{\overline{\pi}_n^C}$, where $\overline{\pi}_n^C$ denotes the long-run average of quarterly CPI inflation in country n. Using quarterly data from 2002Q2 to 2024Q4 and setting $\rho_m^n = 0.95$, we calibrate the inflation response coefficients accordingly. This calibration captures the empirical observation that central banks in many countries outside the United States are less responsive to inflation fluctuations and are therefore less likely to adhere strictly to a Taylor rule. In all countries, we also include a small output weight of $\phi_y^n = 0.1$ to reflect the notion that central banks respond—albeit modestly—to output fluctuations in their policy frameworks.

As the basis for consumption shares and intermediate input shares, we use the OECD Inter-Country Input–Output (ICIO) tables (OECD, 2020) for the year 2019.[21] We aggre-

[20] Boehm et al. (2023) estimate short-run trade elasticities of approximately 0.76 and long-run elasticities around 2. For our tariff scenarios, we adopt the lower short-run elasticity, which better captures the immediate effects that are more relevant for monetary policy. In contrast, USTR (2025) uses a higher value of 4 for the trade elasticity.

[21] Although the latest available data at the time of writing was for 2020, we use 2019 data to avoid

Table 1. Parameter values

Parameter	Explanation	Value	Source
σ	Intertemporal EoS	2	Itskhoki and Mukhin (2021)
γ	Elasticity of Labor	1	Itskhoki and Mukhin (2021)
ψ	Reactivity of UIP to Debt	0.001	Itskhoki and Mukhin (2021)
ρ_m^n	Inertia in Taylor Rule for $n \neq US$	0.95	Clarida et al. (2000)
ρ_m^{US}	Inertia in Taylor Rule for US	0.82	Carvalho et al. (2021)
ϕ_π^{US}	Weight on inflation in Taylor Rule for US	1.29	Carvalho et al. (2021)
λ_n	Sector specific price rigidities		Nakamura and Steinsson (2008)
θ	EoS between intermediates and VA	0.6	Atalay (2017)
σ_h	Intratemporal EoS of consumption among sectors	0.6	Calibrated for consistency
θ_h	EoS among intermediate inputs	0.2	Boehm et al. (2019)
σ_l^i	Sector level consumption bundle EoS	0.6	Di Giovanni et al. (2023)
θ_l^i	Sector level input bundle EoS	0.6	Di Giovanni et al. (2023)

NOTES: "EoS" is the elasticity of substitution.

gate the ICIO data to align with the country and industry groupings used in our analysis. Specifically, we include the United States, euro area, China, Canada, and Mexico—reflecting the countries most affected by the tariff announcements as of April 2025—along with an aggregate entity representing the Rest of the World (RoW). On the industry side, we aggregate sectors into eight broad categories: agriculture, energy, mining, food, manufacturing, electronics, residential services, and other services.

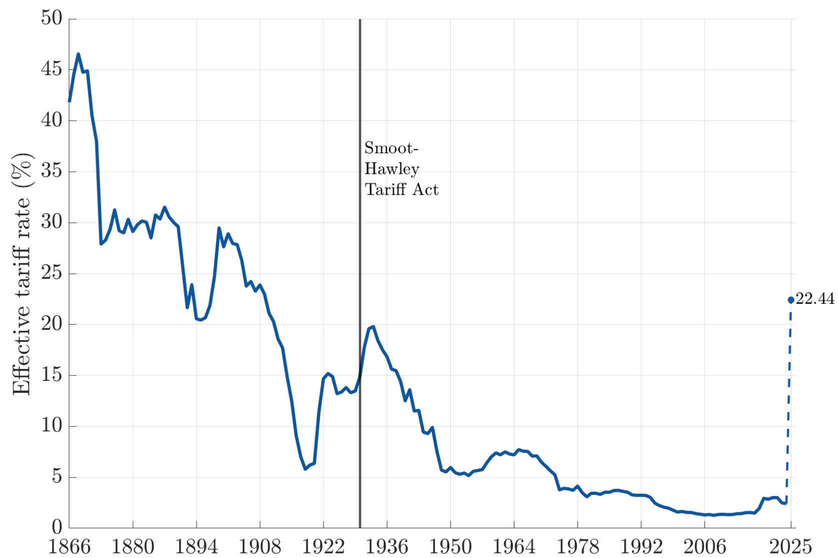
We solve the model with Dynare under three alternative solution methods: first-order approximation, second-order approximation, and MIT shocks under perfect foresight. For small shocks, these methods yield nearly identical impulse response functions. However, our preferred solution approach employs MIT shocks under perfect foresight, given the presence of non-linearities in both the production and consumption structures, as well as the nature of the trade shocks we analyze. We experiment with both permanent (or near-permanent) tariff shocks—modeled as autoregressive processes with coefficients of 0.95 or higher—and transitory shocks, such as one-time tariff increases. While local solution methods (e.g., first-order approximation) are valid only in the neighborhood of the steady state, perfect foresight solutions are better suited for analyzing the effects of permanent shocks that drive the system further from its baseline. Accordingly, for scenarios involving persistent policy changes, the perfect foresight approach provides additional insights beyond what local approximations can offer.

In the exercises that follow, we are motivated by the renewed interest among policymakers in using tariffs as a tool to manage external imbalances and exert geopolitical influence. This interest predates the second Trump presidency and reflects a broader global re-evaluation

distortions arising from the COVID-19 pandemic.

of tariff policy—both for strategic and retaliatory purposes. For contemporary policy relevance, we focus specifically on the tariffs announced in the early months of the second Trump administration. As shown in Figure 4, the tariffs proposed on April 2—referred to as "Liberation Day" by the administration—are projected to raise the effective U.S. tariff rate to 22.4%, the highest level in over a century. This context motivates our analysis of Cases 2, 3, and 4, which examine various implementations and escalations of the 2025 tariff package. These exercises follow the validation of our model using the 2018 U.S.–China Trade War, presented in Case 1.

Figure 4. Effective Tariff Rate (%, Historic and Estimated)

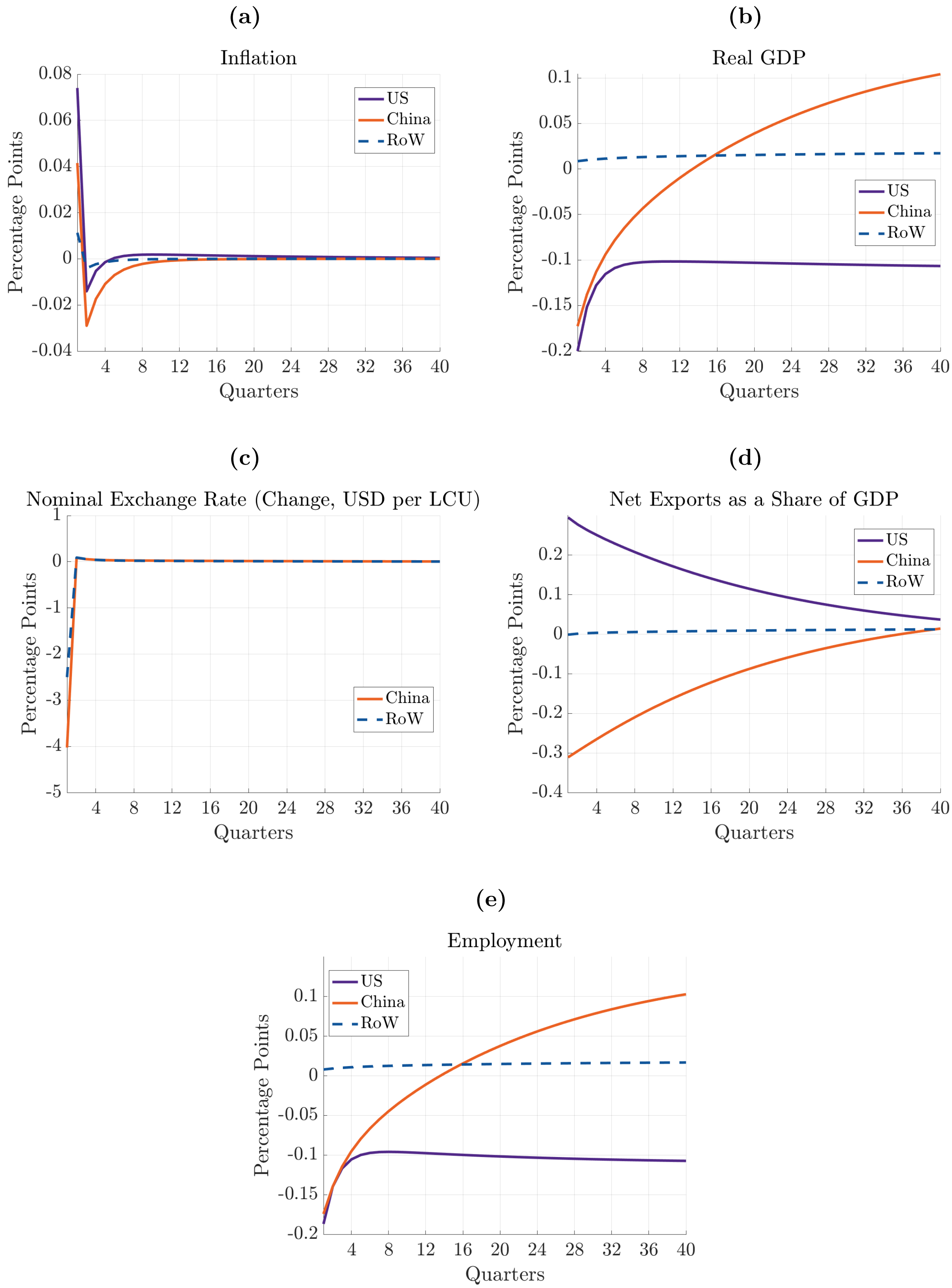


NOTE: Effective tariff rate stands for customs duty revenue as a proportion of goods imports. Data from *Historical Statistics of the United States* Ea424-434, *Monthly Treasury Statement*, Bureau of Economic Analysis. Estimated effective tariff rate of 22.44% provided by Yale Budget Lab using the GTAP Model v7.

4.2 Case 1: 2018's Trade War

We begin by validating the model using the case of the tariffs imposed by the United States on China during 2017–2018. The U.S. implemented a 25% tariff on Chinese imports, which we model as a near-permanent shock with $\rho^{\tau} = 0.95$, assuming no retaliation from China. Deviating from other simulations, based on real-life experiences we assume that the central banks involved did not place a weight on deviation from pre-tariff output (i.e. $phi_y = 0$). As shown in Figure 5, the model predicts a 4% nominal appreciation of the U.S. dollar (USD) against the Chinese yuan. This closely aligns with the observed 5.6% appreciation of the USD between June 2018 and December 2018.

Figure 5. Case 1: Impact of 2018's Trade War



NOTE: Simulated responses to the 2018 U.S. tariff on China. Impulse responses are computed under with MIT shocks, with a near-permanent tariff shock ($\rho^{\tau} = 0.95$). No retaliation is assumed.

The model estimates the impact of the 2018 tariffs on U.S. inflation to be 0.07 percentage points. This is in line with the magnitude of the static estimate of Barbiero and Stein (2025), who find that the tariff war may have contributed between 0.1 and 0.2 percentage points to U.S. PCE inflation. Our estimate lies slightly below the lower end of this range, which is consistent with the structure of our model—featuring nominal rigidities and network complementarities—tending to produce smaller inflationary effects and larger real responses when shocks are realized. On impact, real U.S. GDP declines by 0.2%. This magnitude is comparable to the findings of Fajgelbaum et al. (2020), who estimate that the tariffs resulted in producer and consumer losses totaling 0.4% of GDP. Notably, the model also captures changes in external balances: U.S. net exports increase by 0.3% of steady-state GDP, while China's net exports decline by 0.3%.

Table 2. On-Impact Response of Variables in Case 1: 2018's Trade War

	US	China	RoW
$RGDP_n$	-0.20%	-0.17%	0.01%
C_n	-0.13%	-0.17%	-0.03%
π_n	0.07%	0.04%	0.01%
i_n	0.10%	0.01%	0.00%
$\Delta\mathcal{E}_n$	0.00%	-4.02%	-2.50%
$\Delta NEER_n^{US}$	-2.79%	-2.79%	-2.79%
ΔRER_n	0.00%	-4.06%	-2.56%
L_n	-0.19%	-0.17%	0.01%
$\frac{W_n}{P_n}$	-0.45%	-0.51%	-0.05%
$\frac{NX_n}{NGDP_n}$	0.30%	-0.31%	-0.00%
$\frac{NX_n}{NGDP_n^{ss}}$	0.30%	-0.31%	-0.00%
$\frac{Debt_n}{NGDP_n^{ss}}$	-0.05%	-0.43%	-0.23%

NOTE: First-period impact of 25% U.S. tariffs on Chinese imports under the no-retaliation scenario. Effects are reported in deviation from the steady state. Variables listed here comprise real GDP ($RGDP_n$), real consumption (C_n), consumer price inflation (π_n), interest rate (i_n), depreciation of US nominal exchange rate vis-a-vis country in the column ($\Delta\mathcal{E}_n$), depreciation of the US NEER ($\Delta NEER_n^{US}$), depreciation of the US real exchange rate vis-a-vis country in the column (ΔRER_n), employment (L_n), real wages ($\frac{W_n}{P_n}$), net exports as a share of GDP ($\frac{NX_n}{NGDP_n}$), net exports as a share of steady-state GDP ($\frac{NX_n}{NGDP_n^{ss}}$) and debt as a share of steady-state GDP ($\frac{Debt_n}{NGDP_n^{ss}}$).

China experiences a modest contraction in real GDP, with output declining by 0.2%, accompanied by a 0.2% reduction in consumption and a 0.5% decline in real wages. The renminbi depreciates by 4% in nominal and by 4.1% in real terms.In contrast, the Rest of the World (RoW) experiences a negligible output gain (0.01%), with only minor movements in macroeconomic indicators: inflation rises slightly by 0.01%, the interest rate remains unchanged, and real wages decline by 0.05%.

Figure 5 illustrates the model's dynamics over a ten-year horizon. As shown in Figure 5a, all regions experience an initial inflationary shock, followed by a deflationary adjustment. This underscores how the demand-side component of tariffs can dominate. U.S. real GDP contracts on impact (Figure 5b) and remains approximately 0.1 percentage points below its pre-shock level in the long run. In contrast, China exhibits a gradual recovery, while the Rest of the World (RoW) experiences modest gains, benefiting from the opportunity to substitute for Chinese exports in the U.S. market. Figure 5c displays a post-tariff appreciation of the U.S. dollar relative to both China and the RoW. This realignment is mirrored in trade dynamics: Chinese net exports decline, while U.S. net exports rise (Figure 5d). Employment patterns, shown in Figure 5e, closely follow the path of real GDP. Two factors merit emphasis in the trajectory of Chinese GDP. First, real wages in China decline on impact, stimulating labor demand. This occurs because Chinese monetary policy is less contractionary than that of the United States, where interest rates rise more sharply in response to higher inflation. Second, tariffs function as a negative supply shock for the U.S., causing production to decline alongside demand. As the U.S. increases net exports as a share of GDP, it thereby reduces its debt-to-GDP ratio. China and the RoW partially fill the gap caused by the decline in U.S. production by expanding their own consumption and output.

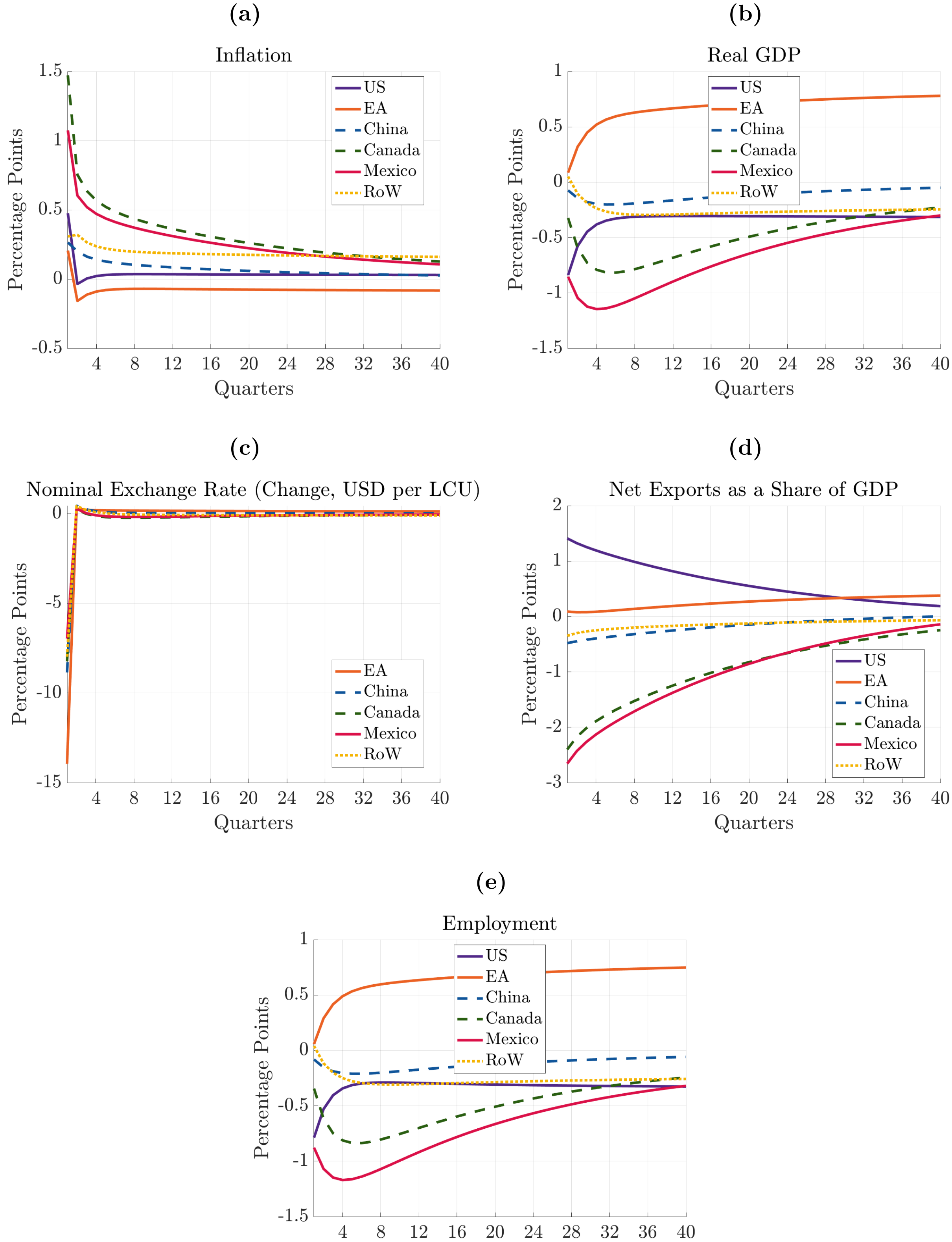
4.3 Case 2: Tariffs Announced on Liberation Day (April 2, 2025)

In February 2025, the United States announced a new round of tariffs targeting Mexico, Canada, and China. Initially, tariffs were set at 25% on imports from Mexico and Canada, and 10% on imports from China. On April 2, 2025, the U.S. government expanded the tariff package, increasing the levy on Chinese imports to 34% and extending tariffs to include the rest of the world. We simulate this scenario by applying tariff rates of 20% on the euro area, 34% on China, 25% on Canada and Mexico, and 10% on the Rest of the World (RoW).[22] To isolate the unilateral effects of U.S. tariff policy, we simulate Case 2 under the assumption of no retaliation and set the tariff persistence parameter to $\rho^{\tau} = 0.95$.

As shown in Figure 6 and Table 3, the model predicts a contraction in U.S. real GDP, declining by 0.8% on impact. This is accompanied by a 1.0% decrease in consumption, a 1.4% increase in net exports as a share of steady-state GDP, and a 2.7% decline in real

[22] While subsequent announcements suggested that Mexico and Canada might be exempt from some or most of the tariffs, for the purpose of this exercise we assume that the United States follows through on its February 2025 declaration and imposes 25% tariffs on both countries without exemptions.

Figure 6. Case 2: Impact of Tariffs Proposed in 2025



Note: Simulated responses to the 2025 U.S. tariff package, targeting China, Canada, Mexico, and the RoW. Impulse responses are computed with MIT shocks with no retaliation and shock persistence of $\rho^{\tau} = 0.95$.

wages. Inflation rises by 0.5 percentage points, prompting a corresponding 0.5 percentage point increase in the nominal interest rate. Additionally, the U.S. trade-weighted nominal effective exchange rate (NEER) appreciates by 10.0%.

The effects are most pronounced for Mexico and Canada. Mexico's real GDP contracts by 0.9%, while Canada's declines by 0.3%. Labor market impacts are also substantial, with employment falling by 0.9% in Mexico and 0.3% in Canada. Net exports decline sharply, by 2.7% and 2.4% of steady-state GDP, respectively. Inflation rises by 1.1 percentage points in Mexico and 1.5 percentage points in Canada.

Table 3. On-Impact Response of Variables in Case 2: 2025's Tariffs

	US	EA	China	Canada	Mexico	RoW
$RGDP_n$	-0.84%	0.09%	-0.07%	-0.32%	-0.85%	0.05%
C_n	-0.95%	-1.41%	0.53%	1.33%	1.44%	0.89%
π_n	0.48%	0.21%	0.27%	1.47%	1.07%	0.31%
i_n	0.54%	0.22%	0.05%	0.26%	0.24%	0.07%
$\Delta\mathcal{E}_n$	0.00%	-13.95%	-8.85%	-8.24%	-6.96%	-7.97%
$\Delta NEER_n^{US}$	-10.02%	-10.02%	-10.02%	-10.02%	-10.02%	-10.02%
ΔRER_n	0.00%	-14.18%	-9.04%	-7.33%	-6.41%	-8.13%
L_n	-0.79%	0.06%	-0.08%	-0.34%	-0.88%	0.04%
$\frac{W_n}{P_n}$	-2.67%	-2.74%	0.99%	2.32%	2.00%	1.82%
$\frac{NX_n}{NGDP_n}$	1.41%	0.09%	-0.48%	-2.40%	-2.66%	-0.35%
$\frac{NX_n}{NGDP_n^{ss}}$	1.42%	0.05%	-0.48%	-2.42%	-2.65%	-0.36%
$\frac{Debt_n}{NGDP_n^{ss}}$	-0.22%	-0.01%	-1.10%	-0.04%	0.11%	-0.73%

NOTE: First-period outcomes of the 2025 unilateral U.S. tariff package under no retaliation. Tariff rates vary by partner; effects are reported in deviation from the steady state. Variables listed here comprise real GDP ($RGDP_n$), real consumption (C_n), consumer price inflation (π_n), interest rate (i_n), depreciation of US nominal exchange rate vis-a-vis country in the column ($\Delta\mathcal{E}_n$), depreciation of the US NEER ($\Delta NEER_n^{US}$), depreciation of the US real exchange rate vis-a-vis country in the column (ΔRER_n), employment (L_n), real wages ($\frac{W_n}{P_n}$), net exports as a share of GDP ($\frac{NX_n}{NGDP_n}$), net exports as a share of steady-state GDP ($\frac{NX_n}{NGDP_n^{ss}}$) and debt as a share of steady-state GDP ($\frac{Debt_n}{NGDP_n^{ss}}$).

China experiences a minimal contraction in both real GDP (–0.1%) and employment (–0.1%). In contrast, consumption increases by 0.5%, and real wages rise by 1.0%. Inflation increases modestly by 0.3 percentage points. Notably, the renminbi depreciates by 8.9% against the U.S. dollar in nominal terms, mitigating adverse trade effects.

The euro area (EA) experiences mild positive output effects, with real GDP increasing by 0.1%. However, this is accompanied by a sharp decline in consumption (–1.4%) and real wages (–2.7%). Inflation in the EA rises by 0.2 percentage points. The Rest of the World similarly records modest output gains (+0.1%), with stronger positive responses in consumption (+0.9%) and real wages (+1.8%), suggesting benefits from trade diversion. Net exports decline slightly (–0.4% of steady-state GDP), while inflation and interest rates increase moderately, by 0.3 and 0.1 percentage points, respectively.

Figure 6 presents the model's dynamic responses to the announced 2025 tariffs, assuming no retaliation from trade partners. As shown in Figure 6a, inflation declines across all regions after the initial period, with both the United States and the Euro Area (EA) experiencing mild deflation. In the medium to long run, only the EA registers a positive effect on real GDP (Figure 6b). This is driven by the fact that the EA faces relatively low tariff rates and is able to capitalize on the decline in US production. Figure 6c shows that the US dollar initially appreciates relative to all other currencies on impact; thereafter there is a small depreciation in the second period, after which the changes in the exchange rate are minimal. In terms of trade balances, Figure 6d shows that net exports improve only slightly for both the US and EA, while all other regions see some deterioration. Employment dynamics, depicted in Figure 6e, closely track real GDP patterns given the household's labor supply decision.

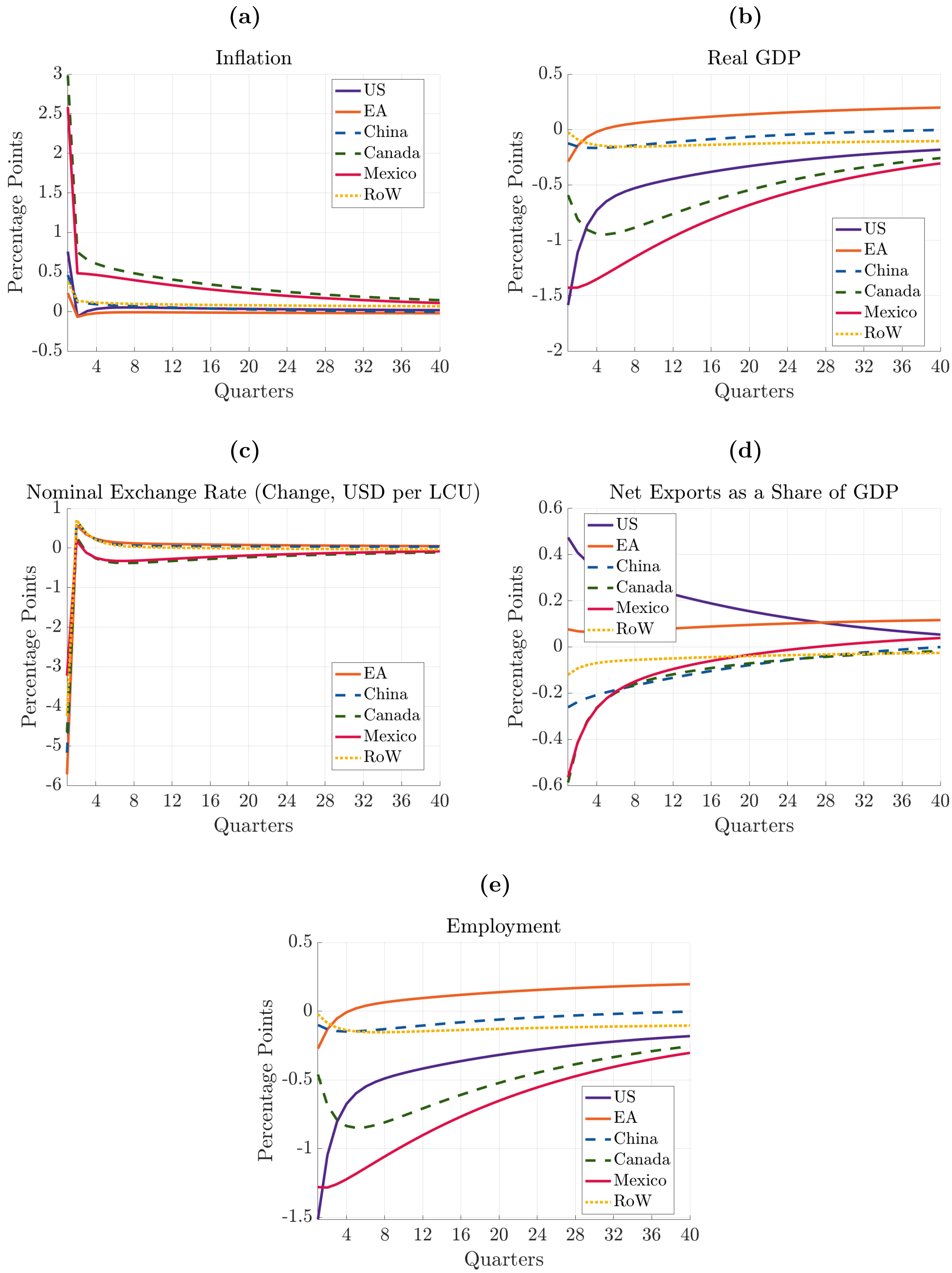
4.4 Case 3: All-Out Trade War

We now turn to a quantitative exercise that mirrors the theoretical simulation presented in Section 3.4, and explore the scenario of an all-out tariff war. In this case, the United States imposes tariffs on all major trade partners at the same rates as specified in Case 2. However, unlike the unilateral shock in Case 2, trade partners retaliate by imposing symmetric tariffs on U.S. exports. The persistence of the tariff shock is set to $\rho^\tau = 0.95$, reflecting a near-permanent policy change.

As illustrated in Figure 7, the model predicts a substantial contraction in U.S. real GDP, which declines by 1.6% on impact. Consumption falls by 1.5%, while net exports increase by 0.5% as a share of steady-state GDP. Inflation rises by 0.8 percentage points, prompting a corresponding increase in the nominal interest rate of 0.8 percentage points. Labor market effects are pronounced, with real wages falling by 4.5% and employment declining by 1.5%. The U.S. NEER appreciates by 4.8%.

The effects of the global tariff war extend across regions, though with heterogeneous intensity. Canada and Mexico are again among the most adversely affected. Real GDP contracts by 0.6% in Canada and by 1.4% in Mexico. Net exports decline by 0.6% of steady-state GDP in both countries, while employment falls by 0.5% in Canada and 1.3%

Figure 7. Case 3: Impact of All-Out Tariff War



Note: All-out tariff war scenario in which trade partners retaliate symmetrically. Impulse responses are calculated with MIT shocks and with shock persistence is set to $\rho^{\tau} = 0.95$. Tariff rates same as Case 2.

in Mexico. Inflation rises sharply—by 3.0 percentage points in Canada and 2.6 percentage points in Mexico. Real wages decline by 1.2% and 2.8%, respectively, indicating substantial labor market strain.

Table 4. On-Impact Response of Variables in Case 3: All-Out Tariff War

	US	EA	China	Canada	Mexico	RoW
$RGDP_n$	-1.58%	-0.29%	-0.12%	-0.59%	-1.43%	-0.03%
C_n	-1.53%	-0.77%	-0.01%	-0.35%	-0.77%	0.18%
π_n	0.76%	0.23%	0.46%	2.98%	2.59%	0.37%
i_n	0.83%	0.21%	0.08%	0.54%	0.63%	0.07%
$\Delta\mathcal{E}_n$	0.00%	-5.72%	-5.17%	-4.66%	-3.22%	-4.24%
$\Delta NEER_n^{US}$	-4.82%	-4.82%	-4.82%	-4.82%	-4.82%	-4.82%
ΔRER_n	0.00%	-6.21%	-5.45%	-2.55%	-1.47%	-4.60%
L_n	-1.51%	-0.27%	-0.10%	-0.46%	-1.28%	-0.02%
$\frac{W_n}{P_n}$	-4.51%	-1.80%	-0.12%	-1.15%	-2.80%	0.34%
$\frac{NX_n}{NGDP_n}$	0.47%	0.08%	-0.26%	-0.59%	-0.56%	-0.12%
$\frac{NX_n}{NGDP_n^{ss}}$	0.51%	0.05%	-0.26%	-0.60%	-0.55%	-0.12%
$\frac{Debt_n}{NGDP_n^{ss}}$	0.02%	-0.01%	-0.68%	-0.04%	0.04%	-0.44%

Note: First-period outcomes from a global tariff war scenario with full retaliation. Tariff magnitudes and persistence match Case 2. Variables listed here comprise real GDP ($RGDP_n$), real consumption (C_n), consumer price inflation (π_n), interest rate (i_n), depreciation of US nominal exchange rate vis-a-vis country in the column ($\Delta\mathcal{E}_n$), depreciation of the US NEER ($\Delta NEER_n^{US}$), depreciation of the US real exchange rate vis-a-vis country in the column (ΔRER_n), employment (L_n), real wages ($\frac{W_n}{P_n}$), net exports as a share of GDP ($\frac{NX_n}{NGDP_n}$), net exports as a share of steady-state GDP ($\frac{NX_n}{NGDP_n^{ss}}$) and debt as a share of steady-state GDP ($\frac{Debt_n}{NGDP_n^{ss}}$).

China experiences a mild contraction in GDP, declining by 0.1%, while consumption remains essentially unchanged (–0.0%). The real exchange rate depreciates by 5.5%. Inflation rises by 0.5 percentage points, and employment declines marginally by 0.1%. Real wages remain largely unaffected, registering a slight decline of 0.1%. The euro area experiences a moderate contraction. Real GDP declines by 0.3%, consumption falls by 0.8%, and real wages decrease by 1.8%. Inflation rises modestly by 0.2 percentage points. The euro depreciates by 5.7% against the U.S. dollar, partly reflecting the divergence in inflation and interest rate responses between the two regions. This exchange rate adjustment helps absorb a portion of the external shock, mitigating further declines in output. The Rest of the World experiences a mild contraction overall. Real GDP remains essentially flat (–0.0%), while consumption rises slightly by 0.2%. Net exports decline marginally by 0.1% of steady-state GDP. Inflation increases by 0.4 percentage points, and real wages rise modestly by 0.3%.

The dynamics of the model under a full trade war, depicted in Figure 7, resemble those in Figure 6, albeit with differences in magnitude. Notably, initial inflation is higher across all regions, while the exchange rate and net export effects are more muted. With the exception of the Euro Area (EA), the levels and trajectories of real GDP and employment closely mirror those observed in the unilateral tariff scenario. However, the EA's gains are smaller under the full trade war, reflecting the broader and more symmetric nature of the global shock. This exercise underscores that retaliation entails significant costs. Although the EA is able to fill the gap left by the decline in US production, it does so to a lesser extent.

As a robustness check, we also examine the implications of a higher Armington elasticity of 4, consistent with the assumption used by USTR (2025). As shown in Figure 11 in Appendix G, the overall qualitative patterns of the results of Case 3 remain broadly unchanged. However, the magnitude of the responses is attenuated under the high-elasticity scenario.

4.5 Case 4: Reversed Tariff Threats

In this section, we apply our model to the case of reversed tariff threats—scenarios in which a country announces future tariffs but subsequently reverses the decision before implementation. This case also incorporates retaliation: specifically, the United States announces in period 1 that tariffs will be imposed in period 2, prompting other countries to announce retaliatory measures for the same period. However, when period 2 arrives, it is announced that no tariffs will be levied by either side. This scenario allows us to isolate the role of the expectations channel and to examine a policy dynamic that has become increasingly common—particularly in the context of U.S. trade policy, where tariff threats are frequently issued and later postponed or rescinded. Our approach is inspired by the *fake news* algorithm of Auclert et al. (2021), in which agents receive information about a future increase in income and optimize accordingly, only to later discover that the anticipated change does not materialize. While Auclert et al. (2021) employ this construct as a computational device for solving models in sequence space, we interpret and apply it literally to study the macroeconomic implications of trade policy reversals.

To analyze the effects of reversed tariff threats, we construct two impulse responses under perfect foresight. First, we simulate the all-out tariff war shock examined in Case 3, assuming it is both announced and implemented in the first period of the simulation. Second, we simulate the same shock—identical in magnitude—but announced to take effect in the second period, only to be withdrawn before implementation. The impulse response to the reversed tariff threat is then obtained by shifting the first (implemented) impulse response forward by one period and subtracting it from the second (announced-but-not-implemented) response. This approach isolates the effect of the anticipatory behavior triggered by the announcement, net of the effects of actual implementation. Importantly, we observe that from the second period onward, the quantity variables in both simulations converge and remain nearly identical. This reflects the fact that agents discount the future and adjust quantities in response to the announcement, but not to the same extent as they would if the shock were immediate and fully realized.

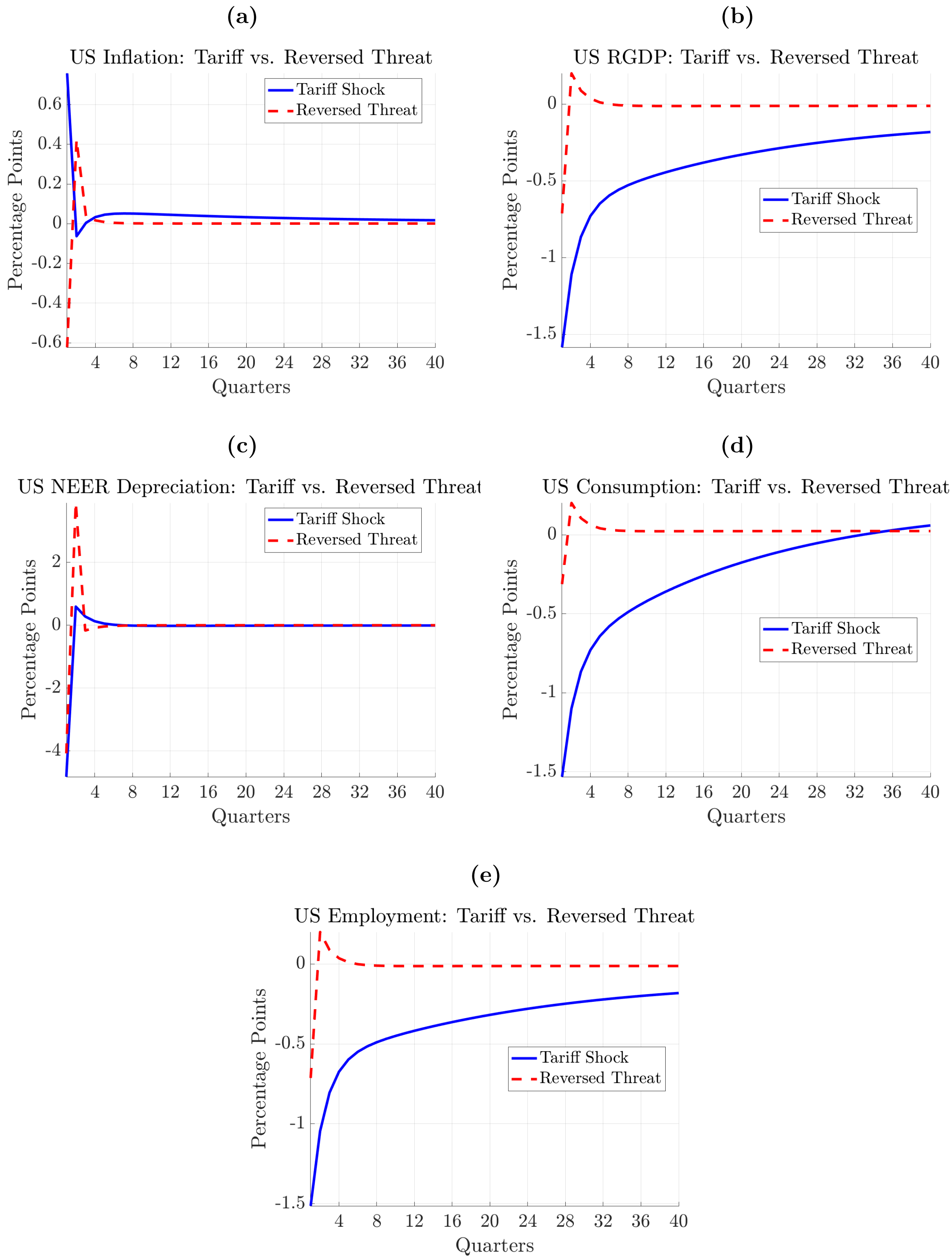
Figure 8 compares the impact of inflation and U.S. dollar appreciation against the Chinese yuan in Case 3 (Tariff Shock) to the reversed tariff threat scenario. Although tariffs are never actually implemented, real variables respond: real GDP and consumption decline by approximately 0.7 and 0.3 percentage points, respectively. Because prices are forward-looking, their responses are of greater magnitude. The near-permanent nature of the anticipated shock induces a pronounced increase in prices, as households and firms adjust their behavior in light of expected future income streams.

A future in which the United States demands fewer goods from China prompts an immediate appreciation of the USD, as agents incorporate these expectations into current pricing. In this scenario, the U.S. trade-weighted nominal effective exchange rate appreciates by 4.1% on impact. In contrast, quantity variables respond more gradually. Consumption declines as households begin smoothing in anticipation of a lower future consumption path. Although consumption begins adjusting toward the level consistent with an immediate tariff shock, it does not fall fully in the first period. When agents realize in the second period that the shock will not materialize, they reoptimize, resulting in a partial recovery. Output follows a similar pattern—declining on impact and gradually recovering thereafter.

Overall, this exercise demonstrates that the expected demand channel, emphasized in our theoretical analysis, plays a central role. Reversed tariff announcements operate similarly to domestic demand shocks, particularly when announcements are perceived as credible. Importantly, the macroeconomic distortion introduced through the expectations channel does not dissipate immediately with the reversal announcement. Variables exhibit persistence, and the economy does not return to steady state instantaneously.

It is notable in the preceding exercise that, once tariffs are reversed, the US dollar depreciates. This outcome is largely mechanical: agents had previously priced in a future in which the US would reduce demand for foreign goods, but upon receiving new information in the second period that this scenario would not materialize, the exchange rate response is reversed. A more realistic interpretation of potential US dollar depreciation in response to tariffs requires accounting for uncertainty and policy volatility, to which we now briefly turn.

Figure 8. Case 4: Impact of Reversed Tariff Threats

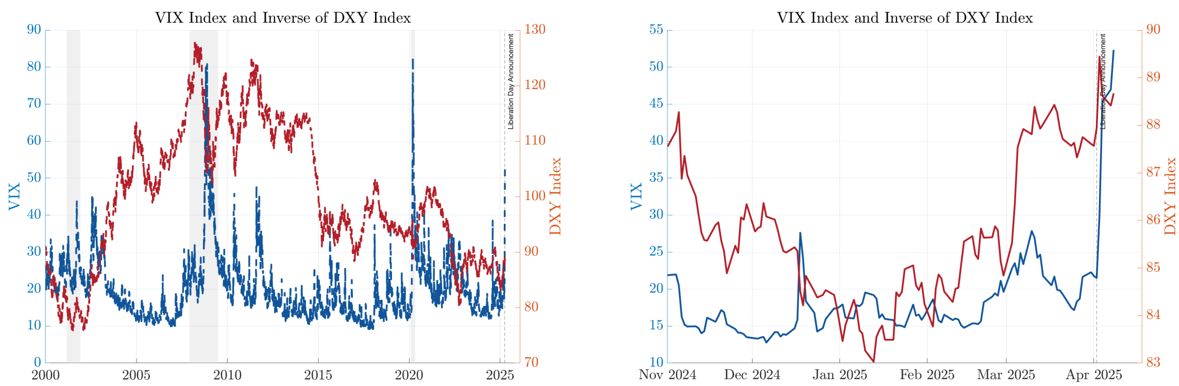


Note: Simulated response to reversed tariff announcements. Tariffs are announced in the first period but canceled in the second period.

As of April 8, 2025, the market response to the “Liberation Day” tariff announcements has been negative, with the VIX index—a standard measure of uncertainty—reaching levels last observed during the COVID-19 pandemic and the Global Financial Crisis, as shown in Figure 9. The figure also indicates that, in prior episodes of elevated volatility and uncertainty, the US dollar functioned as a safe haven, with these periods marked by appreciation in the exchange rate (as a decline in the Inverse DXY index denotes appreciation, consistent with the model’s exchange rate framework). However, because the US is the source of uncertainty in the case of the “Liberation Day” tariffs, it appears that the US dollar may not serve as a safe haven in this instance. Numerous analysts and academics have emphasized that the global shock triggered by the tariff announcements involves not only the direct trade effects, but also heightened uncertainty, which helps explain the adverse market response.

Figure 9. Uncertainty and USD Strength

(a) **(b)**



Note: Figure depicts the VIX Index and the inverse of the DXY Index. The former measures uncertainty and volatility. The DXY Index increases when the dollar is more valuable; in order to make it consistent with the definition of USD strength in our model we invert it so that a decrease corresponds to USD appreciation, while an increase corresponds to USD depreciation. Subfigure (a) depicts the two series from January 2000-onwards, while subfigure (b) focuses on the period after November 2024. Shaded periods corresponds to recession dates defined by NBER.

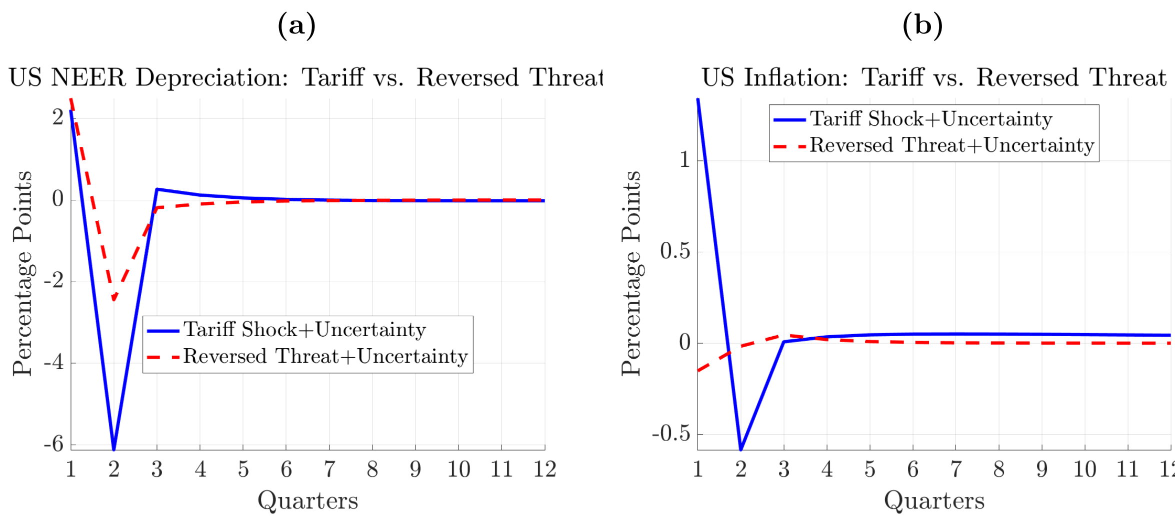
Building on Kalemli-Özcan and Varela (2021) and Itskhoki and Mukhin (2021), Soylu (2025) shows that policy uncertainty can generate a UIP wedge in addition to that arising from financial noise shocks. Accordingly, in the simulation below, we modify the UIP condition as follows:

$$\frac{1+i_{n,t}}{1+i_{n,t}^{US}} = \epsilon^{\tau_t} E_t\left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}}\right] \frac{1}{1-\psi'(B_{n,t}^{US})}$$

where ψ denotes a quadratic loss function targeting the stock of US dollar-denominated debt held by country n, capturing portfolio adjustment costs. Admittedly, this is a reduced-form approach in which the uncertainty associated with tariffs is modeled as a multiplicative shock term, ϵ^{τ_t}. This uncertainty shock enters the UIP condition of all countries vis-à-vis the US and represents a decline in the convenience yield of US assets—or, equivalently, a deterioration in the US's safe haven status. From the perspective of an emerging economy such as Mexico or Turkey (country n), the shock reduces the UIP wedge relative to the US.

The increase in the VIX index following April 1, 2024 corresponds to 3.62 standard deviations. Based on the empirical correlation between the VIX and the UIP premium in the dataset of Kalemli-Özcan and Varela (2021), and consistent with the magnitude of the VIX change between April 1 and April 8, the shock ϵ^{τ_t} is set to $1/(1.081)$ on impact and to zero in all subsequent periods. Figure 10 presents Cases 3 and 4, now incorporating the modified UIP condition with the uncertainty shock entering as a UIP wedge.[23] The uncertainty shock reverses the US dollar appreciation observed in baseline Cases 3 and 4: instead, the US dollar depreciates by 2%, which is similar in magnitude to the 1.3% USD depreciation implied by the DXY index from April 1–8. Importantly, the scenario with uncertainty is associated with higher inflation than the corresponding case without uncertainty.

Figure 10. Cases 3 and 4 Under Uncertainty



NOTE: In these figures we compare Cases 3 and 4 with the UIP condition modified to include an uncertainty shock that serves as a UIP wedge. This shock is imposed (or expected to be imposed) at the same time as tariffs. Case 3 involves "Liberation Day" tariffs. Case 4 involves same tariffs being announced in the first period but canceled in the second period.

[23]Given the structure of the model and implementation in code, the uncertainty shock enters as a UIP wedge in the second period under the reversed threat case; however, agents are informed of the full anticipated path of both tariff and uncertainty shocks in the initial period.

4.6 Discussion

Our analytical and quantitative analyses allow us to engage with several central questions. Under what conditions are tariffs appreciationary or depreciationary for the nominal exchange rate? Under what conditions are tariffs inflationary or deflationary? And under what conditions tariffs can be contractionary?

In our analytical solution, the contemporaneous jump in the nominal exchange rate in response to tariffs is approximately zero. This outcome is driven by home bias and the assumption of a one-time transitory tariff shock, combined with elasticity of substitution parameters below unity—both of which dampen the responsiveness of the balance of payments, as discussed above.

We explore the exchange rate dynamics more fully in Section 4. In our quantitative framework, we find that tariffs can lead to an appreciation of the currency of the tariff-imposing country on impact, as observed in Case 2, and as standard in the literature. However, once retaliation is introduced, the exchange rate response becomes sensitive to the relative hawkishness of central banks. For instance, in a scenario where the U.S. imposes tariffs and the rest of the world responds as in Case 3, the U.S. dollar (USD) may depreciate on impact if the rest of the world has higher ϕ_π parameters—leading to greater interest rate differentials in favor of non-USD currencies. An alternative formulation of Case 3, in which the rest of the world exhibits stronger inflation-targeting behavior than the U.S., confirms this result.

Our analytical work and simulations in Section 4 show that tariffs can be deflationary for the country on which they are imposed, as they reduce demand for that country's goods in the absence of retaliation. A more subtle question is whether tariffs can be deflationary for the tariff-imposing country itself, such as the United States. Within our modeling framework, and barring extreme parameterizations, the direct effect of tariffs, which mechanically exerts upward pressure on prices, dominates the deflationary forces from other channels. If inflation were to turn negative, monetary policy would reverse direction and cut interest rates, thereby supporting prices. Consequently, in both our analytical solution and baseline simulations (Cases 1, 2, and 3), tariffs are inflationary for the imposing country. The key exception is Case 4, in which tariffs are announced to take effect in the following period. In this scenario, using the decomposition developed in Section 3, we observe that the expected demand channel turns negative, while the direct CPI and PPI effects do not materialize at time $t = 0$. This aligns with our simulation results in Case 4, where we indeed find deflation in response to the tariff announcement.

Last, the response of output will depend on the endogenous response of monetary policy to tariff-induced inflation. To explain this further, let us first define the output gap. The standard definition of the output gap in a New Keynesian setting (e.g. $\hat{Y}_t^{\text{gap}} = \hat{Y}_t - \hat{Y}_t^{\text{flex}}$) treats the flexible-price allocation as a moving benchmark. Thus, if a tariff shock occurs, both current output and the flexible-price benchmark are affected. This practice is well-established and has important theoretical advantages, chief among them being divine coincidence. However, our *positive* approach diverges from this standard formulation: our RGDP variable is defined in deviation from the pre-tariff steady state, rather than from a moving flexible-price steady-state output. We adopt this definition because we believe that, in practice—at least in the short run—central banks will place some weight on returning to pre-shock levels of output and employment, to the extent that is possible. This can be both because they believe tariffs are transitory and/or given the uncertainty surrounding tariffs, the new level of potential output is unlikely to be observable. Moreover, this formulation allows us to directly assess claims that tariffs are not recessionary but merely entail a reallocation of demand. Thus, in Cases 2, 3, and 4, when the central bank assigns a weight $\phi_y = 0.1$ to output in its Taylor rule, it does so relative to the pre-tariff steady state. If recent predictions asserting the absence of a recession were accurate, targeting deviations from the pre-tariff output level should not generate a significant permanent inflationary impulse.[24] However, we find this to be the case, confirming the intuition that tariffs act as supply shocks that shift the level of potential output itself. In such a setting, at some point the central bank would have to stop targeting the pre-tariff steady-state output level, as failure to do so would transform permanent tariffs into a persistent cost-push shock, generating a sustained inflationary impulse. This can be observed from the long tails of the IRFs for inflation in Cases 2, 3, and 4.

What about the trade deficit? Our quantitative results confirm that it is possible to improve the U.S. trade balance through the imposition of tariffs. However, our results indicate that this is a costly approach with limited benefits. For example, under retaliation in Case 3, the U.S. trade balance improves by only 0.5% of GDP, while output declines by 1.6% and inflation increases by 0.8%, raising questions about the efficacy of tariffs as a policy tool. A related policy-relevant issue concerns the response of countries targeted by U.S. tariffs. Our quantitative results confirm the intuition that retaliation is costly. A comparison of key macroeconomic indicators between Cases 2 and 3 indicates that countries facing U.S. tariffs would experience greater output losses and higher inflation relative to Case 2 if they were to retaliate symmetrically, however, Europe seems to be the least affected, even under retaliation, given the reallocation of demand across the global network favors

[24] In our network setup, we are aware, as Rubbo (2023) noted, that standard divine coincidence does not hold. Targeting only CPI inflation can, in theory, leave behind a permanent cost-push shock unless one targets the divine coincidence index. In our simulations, however, we find that central bank targeting of only CPI inflation does not result in a major discernible permanent inflationary impulse (e.g. Case 1). This might be because in our model, the CPI shares that are based on the ICIO table happen to be close to the shares that one would assign based on the divine coincidence index proposed by Rubbo (2023).

them as the tariff rates they face are lower.

5 Conclusion

We develop a new global general equilibrium framework to study the endogenous interactions between monetary policy and trade barriers. Our NKOE model incorporates full global input–output linkages, heterogeneity in sectoral price rigidities, and endogenous monetary policy responses to tariffs across all countries involved in a trade war.

The presence of nominal rigidities and production network structure plays a crucial role in shaping the inflation and output responses to tariffs, influencing Phillips Curve dynamics and introducing new wedges relative to the predictions of static trade models. Our quantitative results highlight the inflationary and contractionary effects of tariff shocks in an environment with forward-looking agents, where these effects are further amplified through the expectations channel and exchange rate adjustment. We decompose the general equilibrium response of key macroeconomic variables to trade shocks into channels—each of which maps directly onto structural components of the model. We demonstrate that the net impact of tariffs on domestic inflation and output critically depends on the endogenous monetary policy responses in both the tariff-imposing and tariff-exposed countries, within a global general equilibrium framework.

Our quantitative exercise validates the model by replicating the observed impact of the 2018 tariffs on the U.S. economy and provides forward-looking predictions for a potential trade war associated with the proposed 2025 U.S. tariffs. In the baseline no-retaliation scenario, the model predicts a 0.5 percentage point increase in inflation, a 0.8% decline in output, and a 10% appreciation of the U.S. dollar. Under full retaliation, output declines by 1.6% and inflation rises by 0.8 percentage points. We also find that pure tariff threats—cases in which tariffs are announced but not implemented—are self-defeating. Even in the absence of actual tariffs, the model predicts a 4.1% appreciation of the USD, a 0.6 percentage point decline in inflation (deflation), and a 0.7% drop in output. Furthermore, depending on the relative stance of monetary policy across countries, threats and further increases in global uncertainty can lead to a depreciation of the USD.

Our work yields two main policy implications. First, models that omit a multi-sector structure may underestimate the impact of tariffs on real economic quantities—such as output and employment—while overestimating their effect on inflation. Second, tariff threats carry real macroeconomic consequences—even when they are subsequently reversed. When agents expect future price increases, they begin to smooth consumption downward in anticipation. Because the exchange rate is forward-looking, it appreciates immediately in response to these expectations. In this way, tariff threats function as contractionary demand shocks, even in the absence of actual tariff implementation. Moreover, if the threat originates from the United States, it can induce a meaningful depreciation of the dollar in the future, depending on the relative stance of monetary policy abroad. A deeper understanding of both production network structures and expectation-driven dynamics—such as those modeled here—can help central banks navigate a policy environment in which tariffs, retaliation, and related threats are becoming increasingly common.

By theoretically unifying long- and short-run perspectives on the impact of trade barriers, our framework echoes foundational insights from classical economic literature, dating back to Hume (1758), which emphasized the price–specie flow mechanism. This mechanism illustrates how price levels adjust endogenously through trade flows, ultimately rendering trade restrictions self-defeating. Restrictions on exports and imports induce exchange rate movements that offset any initial perceived gains. For countries imposing import restrictions, rising labor and input costs typically follow, forcing firms to reduce employment and scale back production—ultimately undermining domestic economic performance. This core insight traces back even further to Gervaise (1720), underscoring the long-standing understanding that trade barriers distort price signals and resource allocation.

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A Derivations

A.1 Household's Problem

The Lagrangian for the household's problem is:

$$\begin{aligned}\mathcal{L} = E_0 \Bigg\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{n,t}^{1-\sigma}}{1-\sigma} - \chi \frac{L_{n,t}^{1+\gamma}}{1+\gamma} \right] \\ + \lambda_t \Bigg[\sum_i (W_{n,t} L_{ni,t} + \Pi_{ni,t}) - (1+i_{n,t-1}) B_{n,t-1} - \mathcal{E}_{n,t}(1+i_{n,t-1}^{US}) B_{n,t-1}^{US} \\ -P_{n,t} C_{n,t} - T_{ni,t} + B_{n,t} + \mathcal{E}_{n,t} B_{n,t}^{US} - \mathcal{E}_{n,t}\psi(B_{n,t}^{US}) \Bigg] \Bigg\}.\end{aligned}$$

Given $L_{n,t} = \sum_i L_{ni,t}$, the first-order conditions are:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial C_{n,t}} &= \beta^t C_{n,t}^{-\sigma} - \lambda_t P_{n,t} = 0, \quad \forall t \\ \frac{\partial \mathcal{L}}{\partial L_{n,t}} &= -\beta^t \chi L_{n,t}^{\gamma} + \lambda_t W_{n,t} = 0, \quad \forall t \\ \frac{\partial \mathcal{L}}{\partial B_{n,t}} &= \lambda_t - E_t \lambda_{t+1}(1+i_{n,t}) = 0, \quad \forall t \\ \frac{\partial \mathcal{L}}{\partial B_{n,t}^{US}} &= \lambda_t \mathcal{E}_{n,t} - E_t \lambda_{t+1} \mathcal{E}_{n,t+1}(1+i_{n,t}^{US}) - \lambda_t \mathcal{E}_{n,t} \psi'(B_{n,t}^{US}) = 0, \quad \forall t.\end{aligned}$$

Rearranging the first-order conditions, we derive the key equilibrium conditions.

Euler Equation

Rearranging the FOC for $B_{n,t}$:

$$\lambda_t = E_t \lambda_{t+1}(1+i_{n,t})$$

Substituting $\lambda_t = \frac{\beta^t C_{n,t}^{-\sigma}}{P_{n,t}}$ from the FOC for $C_{n,t}$:

$$1 = \beta E_t \left[\left(\frac{C_{n,t+1}}{C_{n,t}} \right)^{-\sigma} \frac{P_{n,t}}{P_{n,t+1}} (1+i_{n,t}) \right].$$

Intratemporal Labor-Consumption Choice

Rearranging the FOC for $L_{n,t}$:

$$\chi L_{n,t}^{\gamma} = \frac{\lambda_t W_{n,t}}{\beta^t}.$$

Substituting $\lambda_t = \frac{\beta^t C_{n,t}^{-\sigma}}{P_{n,t}}$ from the FOC for $C_{n,t}$:

$$\chi L_{n,t}^{\gamma} = \frac{C_{n,t}^{-\sigma} W_{n,t}}{P_{n,t}}$$
$$\frac{W_{n,t}}{P_{n,t}} = \chi L_{n,t}^{\gamma} C_{n,t}^{\sigma}$$

Uncovered Interest Parity (UIP) Condition with Portfolio Adjustment Costs

Rearranging the FOC for $B_{n,t}^{US}$:

$$\lambda_t \mathcal{E}_{n,t} = E_t \lambda_{t+1} \mathcal{E}_{n,t+1}(1 + i_{n,t}^{US}) + \lambda_t \mathcal{E}_{n,t} \psi'(B_{n,t}^{US}).$$

Dividing both sides by $\lambda_t \mathcal{E}_{n,t}$:

$$1 = E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}} (1 + i_{n,t}^{US}) \right] + \psi'(B_{n,t}^{US}).$$

Using $\lambda_t = E_t \lambda_{t+1}(1 + i_{n,t})$:

$$1 = E_t \left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}} \frac{1 + i_{n,t}^{US}}{1 + i_{n,t}} \right] + \psi'(B_{n,t}^{US})$$
$$\frac{1 + i_{n,t}}{1 + i_{n,t}^{US}} \left(1 - \psi'(B_{n,t}^{US})\right) = E_t \left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}} \right]$$
$$\frac{1 + i_{n,t}}{1 + i_{n,t}^{US}} = E_t \left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}} \right] \frac{1}{1 - \psi'(B_{n,t}^{US})}$$

A.2 Firm Problem

Output in country n sector i at firm f at time t each firm has some CRS production function:

$$Y_{ni,t} = A_{n,i} F_i(L_{ni,t}, \{X_{ni,j,t}\}_{i=1,j=1}^{i=J,j=J})$$

Intermediate goods from different countries are first bundled into a country-industry-good bundle:

$$X_{ni,j,t} = \left[\sum_{m\in N} \beta_{ni,mj}^{\frac{1}{\sigma}} X_{ni,mj,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{46}$$

Relative demand condition is appropriately defined as:

$$X_{ni,mj,t} = \left(\frac{\tau_{n,m,t} P_{n,mj,t}}{P_{ni,j,t}^{p}}\right)^{-\sigma} X_{ni,j,t} \tag{47}$$

where $P_{ni,j,t}^{p}$ is the average price of j for produer sector i in country n.

We next define marginal cost; assuming all firms in a country-sector combination are identical:

$$MC_{ni,t} = \min_{\{X_{ni,j,t}, L_{ni,t}\}} W_t L_{ni,t} + \sum_j P_{ni,j,t}^{p} X_{ni,j,t} \quad \text{s.t.} \quad Y_{ni,t} = 1.$$

Production is CES:

$$Y_{ni,t} = A_{ni,t}\left[\alpha_{ni} L_{ni,t}^{\frac{\sigma-1}{\sigma}} + \sum_j \beta_{ni,j} (X_{ni,j,t})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \forall n \in N, \forall i \in J, \forall m \in N, \forall j \in J \tag{48}$$

This problem yields the following equilibrium conditions:

$$\frac{X_{ni,j,t}}{L_{ni,t}} = \left(\frac{W_t \beta_{ni,j}}{P_{ni,j,t}^{p} \alpha_{ni}}\right)^{\sigma} \forall\ j \in J \tag{49}$$

$$MC_{ni,t} = \frac{1}{A_{ni,t}}\left[\alpha_{ni} W_t^{1-\sigma} + \sum_j \beta_{ni,j} P_{ni,j,t}^{p^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} \tag{50}$$

Setting up the Lagrangian:

$$\mathcal{L} = W_t L_{ni,t} + \sum_j P_{ni,j,t}^{p} X_{ni,j,t} - \lambda_{ni,t}\left(Y_{ni,t} - 1\right)$$

We can solve for the optimal levels given the contraints:

$$\frac{\partial \mathcal{L}}{\partial L_{ni,t}} = W_t - \lambda_{ni,t} \alpha_{ni} L_{ni,t}^{-\frac{1}{\sigma}} A_{ni,t}\left[\alpha_{ni} L_{ni,t}^{\frac{\sigma-1}{\sigma}} + \sum_j \beta_{ni,j} (X_{ni,j,t})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial X_{ni,j,t}} = P^p_{ni,j,t} - \lambda_{ni,t}\beta_{ni,j}(X_{ni,j,t})^{-\frac{1}{\sigma}} A_{ni,t}\left[\alpha_{ni} L_{ni,t}^{\frac{\sigma-1}{\sigma}} + \sum_j \beta_{ni,j}(X_{ni,j,t})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} = 0 \quad \forall j \in J$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{ni,t}} = A_{ni,t}\left[\alpha_{ni} L_{ni,t}^{\frac{\sigma-1}{\sigma}} + \sum_j \beta_{ni,j}(X_{ni,j,t})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - 1 = 0$$

Rearranging the terms:

$$\begin{aligned}
W_t &= \lambda_{ni,t}\alpha_{ni}\left(\frac{Y_{ni_t}}{L_{ni,t}}\right)^{\frac{1}{\sigma}} A_{ni,t}^{\frac{\sigma-1}{\sigma}} \\
P^p_{ni,j,t} &= \lambda_{ni,t}\beta_{ni,j}\left(\frac{Y_{ni_t}}{X_{ni,j,t}}\right)^{\frac{1}{\sigma}} A_{ni,t}^{\frac{\sigma-1}{\sigma}} \quad \forall j \in J \\
\lambda_{ni,t} &= \frac{W_t}{\alpha_{ni} L_{ni,t}^{-\frac{1}{\sigma}}}\left[\alpha_{ni} L_{ni,t}^{\frac{\sigma-1}{\sigma}} + \sum_j \beta_{ni,j}(X_{ni,j,t})^{\frac{\sigma-1}{\sigma}}\right] \forall i \in J
\end{aligned}$$

Further rearranging allows us to substitute out λ_{ni}. Keeping in mind that the MC minimization problem is for $Y_{ni,t} = 1$ and that the production function is homogeneous of degree 1, we can adjust optimal labor choice accordingly. Then we have:

$$\begin{aligned}
\frac{X_{ni,j,t}}{L_{ni,t}} &= \left(\frac{W_t\beta_{ni,j}}{P^p_{ni,j,t}\alpha_{ni}}\right)^{\sigma} \forall\ j \in J \\
L_{ni,t} &= \frac{Y_{ni,t}}{A_{ni,t}}\left(\frac{\alpha_{ni}}{W_t}\right)^{\sigma}\left[\alpha_{ni}^{\sigma} W_t^{1-\sigma} + \sum_j \beta_{ni,j}^{\sigma} P^{p^{1-\sigma}}_{ni,j,t}\right]^{\frac{1}{1-\sigma}} \\
MC_{ni,t} &= \frac{1}{Y_{ni,t}}\left[W_t L_{ni,t} + \sum_j P^p_{ni,j,t} X_{ni,j,t}\right]
\end{aligned}$$

The last expression (plugging in the relative demand terms) can be written as:

$$MC_{ni,t} = \frac{1}{A_{ni,t}}\left[\alpha_{ni} W_t^{1-\sigma} + \sum_j \beta_{ni,j} P^{p^{1-\sigma}}_{ni,j,t}\right]^{\frac{1}{1-\sigma}}$$

A.2.1 Rotemberg Adjustment Costs

Within each country sector there is an infinite continuum of identical firms. Representative firm f in sector i of country n solves the following problem Rotemberg setup:

$$P^f_{ni,t} = \arg\max_{P^f_{ni,t}} \mathbb{E}_t\left[\sum_{T=t}^{\infty} \mathrm{SDF}_{t,T}\left[Y^f_{ni,T}(P^f_{ni,T})\left(P^f_{ni,T} - MC_{ni,T}\right) - \frac{\delta_{ni}}{2}\left(\frac{P^f_{ni,T}}{P^f_{ni,T-1}} - 1\right)^2 Y_{ni,T}P_{ni,T}\right]\right]$$

where a bundler puts together the sectoral output as a CES bundle such that the demand function is $Y^f_{ni,t}(P^f_{ni,t}) = \left(\frac{P^f_{ni,t}}{P_{ni,t}}\right)^{-\theta_l} Y_{ni,t}$. Bundler has log utility; it takes firm-level output and produces sectoral level output and net-zero bond-supply such that the nominal SDF will be $\text{SDF}_{t,T} = \beta^{T-t}\frac{Y_{ni,t}P_{ni,t}}{Y_{ni,T}P_{ni,T}}$. Plugging this in and writing the Lagrangian:

$$\mathcal{L} = \mathbb{E}_t\left[\sum_{T=t}^{\infty}\text{SDF}_{t,T}\left[\left(\frac{P^f_{ni,T}}{P_{ni,T}}\right)^{-\theta_l} Y_{ni,T}\left(P^f_{ni,T} - MC_{ni,T}\right) - \frac{\delta_{ni}}{2}\left(\frac{P^f_{ni,T}}{P^f_{ni,T-1}} - 1\right)^2 Y_{ni,T}P_{ni,T}\right]\right]$$

$$\mathcal{L} = \sum_{T=t}^{\infty}\beta^{T-t}Y_{ni,t}P_{ni,t}\mathbb{E}_t\left[\frac{1}{Y_{ni,T}P_{ni,T}}\left[(P^f_{ni,T})^{1-\theta_l}P_{ni,T}{}^{\theta_l}Y_{ni,T} - \left(\frac{P^f_{ni,T}}{P_{ni,T}}\right)^{-\theta_l} Y_{ni,T}MC_{ni,T} - \frac{\delta_{ni}}{2}\left(\frac{P^f_{ni,T}}{P^f_{ni,T-1}} - 1\right)^2 Y_{ni,T}P_{ni,T}\right]\right]$$

$$\mathcal{L} = \sum_{T=t}^{\infty}\beta^{T-t}Y_{ni,t}P_{ni,t}\mathbb{E}_t\left[(P^f_{ni,T})^{1-\theta_l}(P_{ni,T})^{\theta_l-1} - \left(\frac{P^f_{ni,T}}{P_{ni,T}}\right)^{-\theta_l}\frac{MC_{ni,T}}{P_{ni,T}} - \frac{\delta_{ni}}{2}\left(\frac{P^f_{ni,T}}{P^f_{ni,T-1}} - 1\right)^2\right]$$

Taking the FOC with respect to $P^f_{ni,T}$:

$$\begin{aligned}\frac{\partial Z_t}{\partial P^f_{ni,T}} =& \mathbb{E}_t\left[Y_{ni,t}P_{ni,t}\left[(1-\theta_l)(P^f_{ni,T})^{-\theta_l}(P_{ni,T})^{\theta_l-1} + \theta_l\left(\frac{P^f_{ni,T}}{P_{ni,T}}\right)^{-\theta_l-1}\frac{MC_{ni,T}}{(P_{ni,T})^2} - \delta_{ni}\left(\frac{P^f_{n,T}}{P^f_{n,T-1}} - 1\right)\frac{1}{P^f_{n,T-1}}\right]\right]\\ &+ \beta Y_{ni,t}P_{ni,t}\mathbb{E}_t\left[\delta_{ni}\left(\frac{P^f_{n,T+1}}{P^f_{n,T}} - 1\right)\frac{P^f_{n,T+1}}{(P^f_{n,T})^2}\right] = 0\end{aligned}$$

With $Y_{ni,t}P_{ni,t} \neq 0$ we can divide both sides by $Y_{ni,t}P_{ni,t}$. Additionally firms within an industry are symmetric so $P^f_{n,T} = P_{n,T}$.

$$\begin{aligned}&\mathbb{E}_t\left[(1-\theta_l)(P^f_{ni,T})^{-\theta_l}(P_{ni,T})^{\theta_l-1} + \theta_l\left(\frac{P^f_{ni,T}}{P_{ni,T}}\right)^{-\theta_l-1}\frac{MC_{ni,T}}{(P_{ni,T})^2} - \delta_{ni}\left(\frac{P^f_{n,T}}{P^f_{n,T-1}} - 1\right)\frac{1}{P^f_{n,T-1}}\right]\\ &+ \beta\mathbb{E}_t\left[\delta_{ni}\left(\frac{P^f_{n,T+1}}{P^f_{n,T}} - 1\right)\frac{P^f_{n,T+1}}{(P^f_{n,T})^2}\right] = 0\\ &\mathbb{E}_t\left[(1-\theta_l)P^{-1}_{ni,T} + \theta_l\frac{MC_{ni,T}}{(P_{ni,T})^2} - \delta_{ni}\left(\frac{P_{n,T}}{P_{n,T-1}} - 1\right)\frac{1}{P_{n,T-1}}\right]\\ &+ \beta\mathbb{E}_t\left[\delta_{ni}\left(\frac{P_{n,T+1}}{P_{n,T}} - 1\right)\frac{P_{n,T+1}}{(P_{n,T})^2}\right] = 0\end{aligned}$$

Since T is arbitrary, let us set $t = T$:

$$\left[(1-\theta_l)P^{-1}_{ni,t} + \theta_l\frac{MC_{ni,t}}{(P_{ni,t})^2} - \delta_{ni}\left(\frac{P_{n,t}}{P_{n,t-1}} - 1\right)\frac{1}{P_{n,t-1}}\right] + \beta\mathbb{E}_t\left[\delta_{ni}\left(\frac{P_{n,t+1}}{P_{n,t}} - 1\right)\frac{P_{n,t+1}}{(P_{n,t})^2}\right] = 0$$

Defining gross inflation and multiplying both sides by $\frac{P_{n,t}}{\delta_{ni}}$ and rearranging:

$$\left(\Pi_{ni,t}-1\right)\Pi_{ni,t}=\frac{\theta_l}{\delta_{ni}}\left(\frac{MC_{ni,t}}{P_{ni,t}}-\frac{\theta_l-1}{\theta_l}\right)+\beta\mathbb{E}_t\left[\left(\Pi_{ni,t+1}-1\right)\Pi_{ni,t+1}\right] \tag{51}$$

The FOCs for the MC minimization problem pins down demand for inputs (including labor), so jointly equations (16)-(18) constitute a forward-looking New Keynesian Phillips Curve. As $\delta_{ni}\to 0$ prices are more flexible and we have $\Pi_{n,t}=1$ and $\frac{MC_{ni,t}}{P_{ni,t}}=\frac{\theta_l-1}{\theta_l}$, which is the general pricing equation under monopolistic competition. For the zero inflation steady state where prices are all 1, the equation above can be rewritten as follows:

$$\left(\Pi_{ni,t}-1\right)\Pi_{ni,t}=\frac{\theta_l-1}{\delta_{ni}}\left(\frac{e^{\widehat{MC}_{ni,t}}}{e^{\hat{P}_{ni,t}}}-1\right)+\beta\mathbb{E}_t\left[\left(\Pi_{ni,t+1}-1\right)\Pi_{ni,t+1}\right]$$

B Approximated Linear Equilibrium Conditions

Before simplifications are introduced, linearized equilibrium conditions are as follows:[25]

$$E_t\hat{C}_{n,t+1}-\hat{C}_{n,t}=\frac{1}{\sigma}\left(\hat{i}_t-E_t\pi_{n,t+1}\right) \tag{52}$$

$$\hat{i}_{n,t}-\hat{i}_{US,t}=E_t\hat{\mathcal{E}}_{n,t+1}-\hat{\mathcal{E}}_{n,t}+\hat{\psi} \tag{53}$$

$$\hat{\mathcal{E}}_{n,m,t}=\hat{\mathcal{E}}^{US}_{n,t}-\hat{\mathcal{E}}^{US}_{m,t} \tag{54}$$

$$\hat{\mathcal{E}}_{n,n,t}=0 \tag{55}$$

$$\hat{W}_{n,t}-\hat{P}_{n,t}=\psi\hat{L}_{n,t}+\sigma\hat{C}_{n,t} \tag{56}$$

$$\hat{C}_{nt}=\sum_{j\in J}\Xi_{n,j}\hat{C}_{n,j,t} \tag{57}$$

$$\hat{C}_{n,j,t}=\sum_{m\in N}\Xi_{n,i,mi}\hat{C}_{n,mj,t} \tag{58}$$

$$\hat{P}_{n,mj,t}=\hat{\mathcal{E}}_{n,m,t}+\hat{P}_{mj,t} \tag{59}$$

$$\hat{C}_{n,j,t}=\hat{C}_{n,t}-\sigma_h\left(\hat{P}_{n,j,t}-\hat{P}_{n,t}\right) \tag{60}$$

$$\hat{C}_{n,mj,t}=\hat{C}_{n,j,t}-\sigma_l^j\left(\hat{\tau}_{n,m,t}+\hat{P}_{n,mj,t}-\hat{P}_{n,j,t}\right) \tag{61}$$

$$\hat{X}_{ni,j,t}=\sum_{m\in N}\Omega_{ni,j,mj}\hat{X}_{ni,mj,t} \tag{62}$$

$$\hat{X}_{ni,mj,t}=\hat{X}_{ni,j,t}-\theta_l^j\left(\hat{\tau}_{n,m,t}+\hat{P}_{n,mj,t}-\hat{P}_{ni,j,t}\right) \tag{63}$$

[25] Please note in this set of equilibrium conditions the highest layer of the intermediate input bundle is simplified away.

$$\hat{Y}_{ni,t} = \hat{A}_{ni,t} + \left[\alpha_{ni}\hat{L}_{ni,t} + (1-\alpha_{ni})\sum_j \Omega_{ni,j}\hat{X}_{ni,j,t}\right] \tag{64}$$

$$\widehat{MC}_{ni,t} = -\hat{A}_{ni,t} + \alpha_{ni}\hat{W}_t + (1-\alpha_{ni})\sum_j \Omega_{ni,j}\hat{P}_{ni,j,t} \tag{65}$$

$$\hat{X}_{ni,j,t} - \hat{L}_{ni,t} = \theta_h\hat{W}_t - \theta_h\hat{P}_{ni,j,t} \tag{66}$$

$$\pi_{ni,t} = \frac{\theta_r}{\delta_{ni}}\left(\widehat{MC}_{ni,t} - \hat{P}_{ni,t}\right) + \beta\mathbb{E}_t\pi_{ni,t+1} \tag{67}$$

$$\bar{B}^{US}\hat{B}_t^{US} = \sum_m^{N-1} \bar{B}_m^{US}\hat{B}_{m,t}^{US} \tag{68}$$

$$\bar{Y}_{ni}\hat{Y}_{ni,t} = \sum_{n\in\mathcal{N}} \bar{C}_{m,ni}\hat{C}_{m,ni,t} + \sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{X}_{mj,ni}\hat{X}_{mj,ni,t}, \tag{69}$$

$$\bar{L}_n\hat{L}_{n,t} = \sum_{i\in J} \bar{L}_{ni}\hat{L}_{ni,t} \tag{70}$$

$$\pi_{n,t} = \hat{P}_{n,t} - \hat{P}_{n,t-1} \tag{71}$$

$$\hat{i}_{n,t} = \phi_\pi\pi_{n,t} + \hat{M}_{n,t} \tag{72}$$

and

$$\begin{aligned}
&\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{C}_{n,mj}(\hat{P}_{n,mj,t} + \hat{C}_{n,mj,t}) + \sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{X}_{ni,mj}(\hat{P}_{n,mj,t} + \hat{X}_{ni,mj,t}) \\
&+ \bar{\mathcal{E}}_n(1+\bar{i}_n^{US})\bar{B}_n^{US}\left(\hat{\mathcal{E}}_{n,t} + \hat{i}_{n,t-1}^{US} + \hat{B}_{n,t-1}^{US}\right) \\
=&\sum_i \bar{P}_{ni}\bar{Y}_{ni}(\hat{P}_{ni,t} + \hat{Y}_{ni,t}) + \bar{\mathcal{E}}_n\bar{B}_n^{US}(\hat{\mathcal{E}}_{n,t} + \hat{B}_{n,t}^{US})
\end{aligned} \tag{73}$$

C Analytical Solution with Fixed Nominal Demand

C.1 NKPC

Recalling producer inflation:

$$\pi_{ni,t}^p = \frac{\theta_l}{\delta_{ni}}\left(\alpha_{ni}\underbrace{\hat{W}_{n,t}}_{\hat{M}_{n,t}} + \sum_{m\in N}\sum_{j\in J}\omega_{ni,mj}(\hat{P}_{mj,t}^p + \underbrace{\hat{\mathcal{E}}_{n,m,t}}_{\hat{M}_{n,t}-\hat{M}_{m,t}} + \tau_{n,mj,t}) - \hat{P}_{ni,t}^p\right) + \beta_n\mathbb{E}_t\pi_{ni,t+1}^p$$

Then in vector and matrix notation we have:

$$\underbrace{\boldsymbol{\pi}_t^P}_{NJ\times 1} = \underbrace{\boldsymbol{\Lambda}}_{NJ\times NJ}\Bigg(\underbrace{\boldsymbol{\alpha}}_{NJ\times NJ}\underbrace{\hat{\boldsymbol{M}}_t}_{NJ\times 1} + \underbrace{(\boldsymbol{\Omega}-\boldsymbol{I})}_{NJ\times NJ}\underbrace{\hat{\boldsymbol{P}}_t^P}_{NJ\times 1} + \underbrace{\underbrace{[\boldsymbol{\Omega}}_{NJ\times NJ}\odot\underbrace{\hat{\boldsymbol{\mathcal{E}}}_t]}_{NJ\times NJ}\underbrace{\mathbf{1}}_{NJ\times 1}}$$

$$+\underbrace{[\mathbf{\Omega}}_{NJ\times NJ}\odot\underbrace{\hat{\boldsymbol{\tau}}_t]}_{NJ\times NJ}\underbrace{\mathbf{1}}_{NJ\times 1}\Bigg)+\underbrace{\boldsymbol{\beta}}_{NJ\times NJ}\mathbb{E}_t\underbrace{\boldsymbol{\pi}^P_{t+1}}_{NJ\times 1},$$

where α is the diagonal matrix whose non-zero elements are the labor-shares (i.e., α_{ni}) and $\hat{\boldsymbol{M}}_t$ is a $NJ\times 1$ vector such that $\hat{\boldsymbol{M}}_{ni,t}=\hat{M}_{n,t}$. We will use the following Lemma to simplify the equations.

Lemma 1. *Given $\hat{\boldsymbol{\mathcal{E}}}_{ni,mj,t}=\hat{\boldsymbol{M}}_{ni,t}-\hat{\boldsymbol{M}}_{mj,t}$, we can write:*

$$[\mathbf{\Omega}\odot\hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1}=(\boldsymbol{I}-\boldsymbol{\alpha}-\mathbf{\Omega})\hat{\boldsymbol{M}}_t$$

The proof follows from calculating each element:

$$\sum_{mj}\Omega_{ni,mj}\hat{\boldsymbol{\mathcal{E}}}_{ni,mj,t}=\sum_{mj}\Omega_{ni,mj}(\hat{\boldsymbol{M}}_{ni,t}-\hat{\boldsymbol{M}}_{mj,t})=\hat{\boldsymbol{M}}_{ni,t}\underbrace{\sum_{mj}\Omega_{ni,mj}}_{1-\alpha_{ni}}-\sum_{mj}\Omega_{ni,mj}\hat{\boldsymbol{M}}_{mj,t}.$$

Lemma 2. *We can write:*

$$[\mathbf{\Omega}\odot\hat{\boldsymbol{\tau}}_t]\mathbf{1}=\breve{\mathbf{\Omega}}\breve{\boldsymbol{\tau}}_t,$$

where $\breve{\mathbf{\Omega}}$ is a $NJ\times(N\times NJ)$ block diagonal matrix with:

$$\breve{\mathbf{\Omega}}_{ni,l*(NJ-1)+mj}=\begin{cases}\mathbf{\Omega}_{ni,mj} & \text{if}\quad l=n\\ 0 & \text{otherwise}\end{cases}$$

and $\breve{\boldsymbol{\tau}}_t$ is a $(N\times NJ)\times 1$ vector whose elements are given by $\breve{\boldsymbol{\tau}}_{n*(NJ-1)+mj,t}=\hat{\tau}_{n,mj,t}$.

This equality can be seen easily by calculating the summations. Therefore, we can write the producer inflation as:

$$\boldsymbol{\pi}^P_t=\mathbf{\Lambda}\Bigg([\mathbf{\Omega}-\boldsymbol{I}]\hat{\boldsymbol{P}}^P_t+[\boldsymbol{I}-\mathbf{\Omega}]\hat{\boldsymbol{M}}_t+\breve{\mathbf{\Omega}}\breve{\boldsymbol{\tau}}_t\Bigg)+\boldsymbol{\beta}\mathbb{E}_t\boldsymbol{\pi}^P_{t+1}.$$

C.2 Method of Undetermined Coefficients

Rewriting (33) purely in terms of the price level as follows, we can solve it analytically:

$$\boldsymbol{\pi}^P_t=\mathbf{\Lambda}\Bigg([\mathbf{\Omega}-\boldsymbol{I}]\hat{\boldsymbol{P}}^P_t+[\boldsymbol{I}-\mathbf{\Omega}]\hat{\boldsymbol{M}}_t+\breve{\mathbf{\Omega}}\breve{\boldsymbol{\tau}}_t\Bigg)+\boldsymbol{\beta}\mathbb{E}_t\boldsymbol{\pi}^P_{t+1}$$

$$(\hat{\boldsymbol{P}}^P_t-\hat{\boldsymbol{P}}^P_{t-1})=\mathbf{\Lambda}\Bigg([\mathbf{\Omega}-\boldsymbol{I}]\hat{\boldsymbol{P}}^P_t+[\boldsymbol{I}-\mathbf{\Omega}]\hat{\boldsymbol{M}}_t+\breve{\mathbf{\Omega}}\breve{\boldsymbol{\tau}}_t\Bigg)+\boldsymbol{\beta}\mathbb{E}_t\left(\hat{\boldsymbol{P}}^P_{t+1}-\hat{\boldsymbol{P}}^P_t\right)$$

$$(\boldsymbol{I}+\boldsymbol{\beta}-\boldsymbol{\Lambda}[\boldsymbol{\Omega}-\boldsymbol{I}])\,\hat{\boldsymbol{P}}_t^P=\hat{\boldsymbol{P}}_{t-1}^P+\boldsymbol{\beta}\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P+\boldsymbol{\Lambda}\left([\boldsymbol{I}-\boldsymbol{\Omega}]\hat{\boldsymbol{M}}_t+\breve{\boldsymbol{\Omega}}\breve{\boldsymbol{\tau}}_t\right)$$

Then:

$$\begin{aligned}
\hat{\boldsymbol{P}}_t^P &= \underbrace{(\boldsymbol{I}+\boldsymbol{\beta}+\boldsymbol{\Lambda}-\boldsymbol{\Lambda}\boldsymbol{\Omega})^{-1}}_{\tilde{\boldsymbol{\Psi}}}\left[\hat{\boldsymbol{P}}_{t-1}^P+\boldsymbol{\beta}\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P+\underbrace{\boldsymbol{\Lambda}[\boldsymbol{I}-\boldsymbol{\Omega}]}_{\boldsymbol{A}}\hat{\boldsymbol{M}}_t+\underbrace{\boldsymbol{\Lambda}\breve{\boldsymbol{\Omega}}}_{\boldsymbol{B}}\breve{\boldsymbol{\tau}}_t\right]\\
\boldsymbol{P}_t^P &= \tilde{\boldsymbol{\Psi}}\boldsymbol{A}\hat{\boldsymbol{M}}_t+\tilde{\boldsymbol{\Psi}}\boldsymbol{B}\breve{\boldsymbol{\tau}}_t+\tilde{\boldsymbol{\Psi}}\hat{\boldsymbol{P}}_{t-1}^P+\tilde{\boldsymbol{\Psi}}\boldsymbol{\beta}(\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P)
\end{aligned}$$

We next do a manipulation to find a system where the matrix on the lagged vector is diagonal. To do so we diagonalize $\tilde{\boldsymbol{\Psi}}$. Defining:[26]

$$\begin{aligned}
\tilde{\boldsymbol{\Psi}} &= \boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}\\
\tilde{\boldsymbol{P}}_t^P &= \boldsymbol{Q}^{-1}\hat{\boldsymbol{P}}_t^P\\
\tilde{\boldsymbol{A}} &= \boldsymbol{Q}^{-1}\boldsymbol{A}\\
\tilde{\boldsymbol{B}} &= \boldsymbol{Q}^{-1}\boldsymbol{B}\\
\tilde{\boldsymbol{\beta}} &= \boldsymbol{Q}^{-1}\boldsymbol{\beta}\boldsymbol{Q}
\end{aligned}$$

Multiplying both sides on the left by $\boldsymbol{Q}^{-1}$ we have:

$$\begin{aligned}
\boldsymbol{P}_t^P &= \tilde{\boldsymbol{\Psi}}\boldsymbol{A}\hat{\boldsymbol{M}}_t+\tilde{\boldsymbol{\Psi}}\boldsymbol{B}\breve{\boldsymbol{\tau}}_t+\tilde{\boldsymbol{\Psi}}\hat{\boldsymbol{P}}_{t-1}^P+\tilde{\boldsymbol{\Psi}}\boldsymbol{\beta}(\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P)\\
\boldsymbol{Q}^{-1}\boldsymbol{P}_t^P &= \boldsymbol{Q}^{-1}\boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}\boldsymbol{A}\hat{\boldsymbol{M}}_t+\boldsymbol{Q}^{-1}\boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}\boldsymbol{B}\breve{\boldsymbol{\tau}}_t+\boldsymbol{Q}^{-1}\boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}\hat{\boldsymbol{P}}_{t-1}^P+\boldsymbol{Q}^{-1}\boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}\boldsymbol{\beta}\boldsymbol{Q}\boldsymbol{Q}^{-1}(\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P)\\
\tilde{\boldsymbol{P}}_t^P &= \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{A}}\hat{\boldsymbol{M}}_t+\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{B}}\breve{\boldsymbol{\tau}}_t+\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{P}}_{t-1}^P+\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}(\mathbb{E}_t\tilde{\boldsymbol{P}}_{t+1}^P)
\end{aligned}$$

Now we have the coefficient on the lag and forward price vector being diagonal, which will come in handy. We can next postulate:

$$\begin{aligned}
\tilde{\boldsymbol{P}}_t^P &= \boldsymbol{C}_1\hat{\boldsymbol{M}}_t+\boldsymbol{C}_2\breve{\boldsymbol{\tau}}_t+\boldsymbol{C}_3\tilde{\boldsymbol{P}}_{t-1}^P\\
\mathbb{E}\tilde{\boldsymbol{P}}_{t+1}^P &= \boldsymbol{C}_3\tilde{\boldsymbol{P}}_t^P=\boldsymbol{C}_3\boldsymbol{C}_1\hat{\boldsymbol{M}}_t+\boldsymbol{C}_3\boldsymbol{C}_2\breve{\boldsymbol{\tau}}_t+\boldsymbol{C}_3\boldsymbol{C}_3\tilde{\boldsymbol{P}}_{t-1}^P
\end{aligned}$$

Plugging these into the expression above:

$$\begin{aligned}
\boldsymbol{C}_1\hat{\boldsymbol{M}}_t+\boldsymbol{C}_2\breve{\boldsymbol{\tau}}_t+\boldsymbol{C}_3\tilde{\boldsymbol{P}}_{t-1}^P &= \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{A}}\hat{\boldsymbol{M}}_t+\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{B}}\breve{\boldsymbol{\tau}}_t+\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{P}}_{t-1}^P+\tilde{\boldsymbol{\beta}}\breve{\boldsymbol{\Psi}}\left(\boldsymbol{C}_3\boldsymbol{C}_1\hat{\boldsymbol{M}}_t+\boldsymbol{C}_3\boldsymbol{C}_2\breve{\boldsymbol{\tau}}_t+\boldsymbol{C}_3\boldsymbol{C}_3\tilde{\boldsymbol{P}}_{t-1}^P\right)\\
\left(\boldsymbol{C}_1-\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{A}}-\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3\boldsymbol{C}_1\right)\hat{\boldsymbol{M}}_t &+ \left(\boldsymbol{C}_2-\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{B}}-\tilde{\boldsymbol{\beta}}\breve{\boldsymbol{\Psi}}\boldsymbol{C}_3\boldsymbol{C}_2\right)\breve{\boldsymbol{\tau}}_t+\left(\boldsymbol{C}_3-\breve{\boldsymbol{\Psi}}-\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3\boldsymbol{C}_3\right)\tilde{\boldsymbol{P}}_{t-1}^P=\boldsymbol{0}
\end{aligned}$$

[26]it is important to note that $\tilde{\boldsymbol{\Psi}}$ is almost diagonal to begin with. Hence, in an approximation sense this step might not be needed.

We have a system of three matrix equations and three unknown matrices ($\boldsymbol{C}_1$,$\boldsymbol{C}_2$,$\boldsymbol{C}_3$):

$$\begin{gathered}\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{A}} - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3\boldsymbol{C}_1 = \boldsymbol{0} \rightarrow \boldsymbol{C}_1 = (\boldsymbol{I} - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3)^{-1}\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{A}} \\ \boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{B}} - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3\boldsymbol{C}_2 = \boldsymbol{0} \rightarrow \boldsymbol{C}_2 = (\boldsymbol{I} - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3)^{-1}\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{B}} \\ \boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}} - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3\boldsymbol{C}_3 = \boldsymbol{0} \rightarrow \boldsymbol{C}_3 = (\boldsymbol{I} - \breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3)^{-1}\breve{\boldsymbol{\Psi}}\end{gathered}$$

Hence, we can solve $\boldsymbol{C}_3$ and then plug it into other coefficients.

$$\breve{\boldsymbol{\Psi}}\tilde{\boldsymbol{\beta}}\boldsymbol{C}_3^2 - \boldsymbol{C}_3 + \breve{\boldsymbol{\Psi}} = \boldsymbol{0}$$

If we assume all discount factors are the same for the countries, i.e., $\beta_n = \beta \quad \forall n$, $\tilde{\boldsymbol{\beta}}$ becomes $\beta\boldsymbol{I}$. Hence, $\boldsymbol{C}_3$ will be diagonal like $\breve{\boldsymbol{\Psi}}$, so we can solve for its diagonal elements explicitly. Since $\boldsymbol{C}_3$ and $\breve{\boldsymbol{\Psi}}$ are diagonal, let their ith diagonal elements be $C_{3,i}$ and $\breve{\Psi}_i$, respectively. The quadratic equation for each diagonal element is:

$$\beta\breve{\Psi}_i C_{3,i}^2 - C_{3,i} + \breve{\Psi}_i = 0$$

Solving for $C_{3,i}$:

$$C_{3,i} = \frac{1 \pm \sqrt{1 - 4\beta\breve{\Psi}_i^2}}{2\beta\breve{\Psi}_i}$$

Since $\boldsymbol{C}_3$ is diagonal, it is constructed as:

$$\boldsymbol{C}_3 = \mathrm{diag}(C_{3,1}, C_{3,2}, \ldots, C_{3,n})$$

where each $C_{3,i}$ is obtained from the quadratic solution above. Given stability requirements, we select the root that satisfies $|C_{3,i}| \leq 1$, ensuring the process does not diverge.[27]

In effect $\tilde{\boldsymbol{\Psi}}$ is already almost diagonal so $\tilde{\boldsymbol{\Psi}}$ is numerically very close to being the identity matrix. For that reason, going forward we will simplify away the tilde notation.

$$\begin{gathered}\boldsymbol{C}_1 = \boldsymbol{C}_3\tilde{\boldsymbol{A}} = \boldsymbol{C}_3\boldsymbol{Q}^{-1}\boldsymbol{A} \\ \boldsymbol{C}_2 = \boldsymbol{C}_3\tilde{\boldsymbol{\Lambda}} = \boldsymbol{C}_3\boldsymbol{Q}^{-1}\boldsymbol{\Lambda}\end{gathered}$$

Let us call the matrix that is constructed to transform $\boldsymbol{C}_3$ back to the industry coordinates with $\boldsymbol{\rho} = \boldsymbol{Q}\boldsymbol{C}_3\boldsymbol{Q}^{-1}$. Thus, substituting these into our expression for $\boldsymbol{P}_t^P$:

$$\boldsymbol{P}_t^P = \boldsymbol{\rho}(\boldsymbol{A}\hat{\boldsymbol{M}}_t + \boldsymbol{\Lambda}\breve{\boldsymbol{\tau}}_t + \boldsymbol{P}_{t-1}^P)$$

[27] We allow for the price *level* can have persistence in the long-run; hence the weak inequality.

Substituting for $\boldsymbol{A}$, and $\boldsymbol{B}$ and subtracting $\boldsymbol{P}^P_{t-1}$ from both sides:

$$\begin{aligned}\boldsymbol{\pi}^P_t &= \boldsymbol{\rho\Lambda}[\boldsymbol{I}-\boldsymbol{\Omega}]\hat{\boldsymbol{M}}_t + \boldsymbol{\rho\Lambda}\breve{\boldsymbol{\Omega}}\breve{\boldsymbol{\tau}}_t + (\boldsymbol{\rho}-\boldsymbol{I})\boldsymbol{P}^P_{t-1} \\ &= \boldsymbol{\rho\Lambda}[\boldsymbol{I}-\boldsymbol{\Omega}]\hat{\boldsymbol{M}}_t + \boldsymbol{\rho\Lambda}[\boldsymbol{\Omega}\odot\hat{\boldsymbol{\tau}}_t]\boldsymbol{1} + (\boldsymbol{\rho}-\boldsymbol{I})\boldsymbol{P}^P_{t-1}\end{aligned}$$

Similar to Blanchard-Kahn conditions we need the solution that ensures all the eigenvalues of $\boldsymbol{\rho}$ are inside the unit circle.

Now let's assume that the countries increase their tariffs with the same amount $\hat{\tau}_{n,mi,t} = \hat{\tau}$ with $\hat{\tau}_{n,ni,t} = 0, \quad \forall n, ni, mi$, since there are no tariffs domestically. With these assumptions,

$$\breve{\boldsymbol{\Omega}}\breve{\boldsymbol{\tau}}_t = \tilde{\boldsymbol{\Omega}}^F\hat{\tau}$$

where $\tilde{\boldsymbol{\Omega}}^F$ is an $NJ\times 1$ dimensional vector that represent the foreign weight in the inputs, respectively. Hence, the impact of a one-time tariff on the producer price inflation vector under price stickiness is:

$$\frac{\partial\boldsymbol{\pi}^P_t}{\partial\tau_t} = \boldsymbol{\rho\Lambda}\tilde{\boldsymbol{\Omega}}^F$$

where $\tilde{\boldsymbol{\Omega}}^F$ is a $NJ\times 1$ vector whose elements are the row sum of the foreign elements of $\boldsymbol{\Omega}$.

D Analytical Solution under $\phi_\pi \to 1$

D.1 Forwarding the Euler Equation

Plug in the Taylor Rule and assume $\sigma = 1$, we have:

$$\hat{C}_{n,t} = E_t\hat{C}_{nt+1} - (\phi^n_\pi\pi_{n,t} - E_t\pi_{n,t+1})$$

Forwarding this we can write today's consumption as the sum of future expected real rates, which in turn can be expressed in terms of inflation differentials, under the assumption that $\lim_{t\to\infty}\hat{C}_{n,t} = 0$:

$$\hat{C}_{n,t} = -E_t\sum_{j=0}^{\infty}[\phi^n_\pi\pi_{n,t+j} - \pi_{n,t+j+1}] = -\phi^n_\pi\pi_{n,t} + (1-\phi^n_\pi)E_t\sum_{j=1}^{\infty}\pi_{n,t+j}$$

Taking the limit of $\phi_\pi \to 1$:

$$\hat{C}_{n,t} = -\pi_{n,t} \tag{74}$$

Our simulations confirm that Equation (74) is identical to the standard Euler equation as $\phi_\pi \to 1$. The intuition is that as inflation rises, central bank will raise rates (and even if it only infinitesimally raises the real rate) that will reduce consumption. More broadly we are deriving an aggregate demand curve that is downward sloping in inflation and can be written as a contemporaneous equation.

This is similar in spirit to fixing nominal demand with $M_{n,t} = P_{n,t}C_{n,t}$; however, this allows for there to be fluctuation in both the nominal and real exchange rates. In general this setup makes it easier to see the feedback loop from prices to demand as opposed to approaches that fix consumption and make it almost exogenous.

In our analytical work instead of taking the limit to 1, we will assume $\phi_\pi \approx 1$ such that we write (74) as follows:

$$\hat{C}_{n,t} \approx -\phi_\pi \pi_{n,t}$$

Numerically this serves as an accurate approximation when $\phi_\pi \approx 1$ and when the shocks at hand are transitory.

D.2 Solving the Exchange Rate

Simplifying away the stationarity inducing device of portfolio adjustment costs, the UIP condition is:

$$\hat{i}_{n,t} - \hat{i}_{m,t} = E_t \hat{\mathcal{E}}_{n,m,t+1} - \hat{\mathcal{E}}_{n,m,t}$$

Rearranging:

$$\hat{\mathcal{E}}_{n,m,t} = E_t \hat{\mathcal{E}}_{n,m,t+1} - (\hat{i}_{n,t} - \hat{i}_{m,t})$$

Plugging in policy rule:

$$\hat{\mathcal{E}}_{n,m,t} = E_t \hat{\mathcal{E}}_{n,m,t+1} + (\phi_\pi^m \pi_{m,t} - \phi_\pi^n \pi_{n,t})$$

Forwarding:

$$\hat{\mathcal{E}}_{n,m,t} = \overline{\mathcal{E}}_{n,m} + \ \phi_\pi E_t \left[\sum_{j=0}^{\infty} (\phi_\pi^m \pi_{m,t+j} - \phi_\pi^n \pi_{n,t+j}) \right]$$

where $\overline{\mathcal{E}}_{n,m} = \lim_{t\to\infty} \hat{\mathcal{E}}_{n,m,t}$ (i.e., we allow for the nominal exchange rate to settle at a permanently different level after shocks as opposed to requiring all nominal variables to return to steady state- real variables will do so).

Defining the real exchange rate between countries and its first difference:

$$\hat{Q}_{m,n,t} = \hat{P}_{m,t} + \hat{\mathcal{E}}_{n,m,t} - \hat{P}_{n,t} \tag{75}$$

$$\Delta\hat{Q}_{m,n,t} = \pi_{m,t} + \Delta\hat{\mathcal{E}}_{n,m,t} - \pi_{n,t} \tag{76}$$

Recalling the Backus Smith condition:

$$\sigma\left(E_t\Delta\hat{C}_{n,t+1} - E_t\Delta\hat{C}_{m,t+1}\right) = E_t\Delta\hat{Q}_{n,m,t+1}$$

Plugging in $\hat{C}_{n,t} = -\pi_{n,t}$ and $\hat{C}_{m,t} = -\pi_{m,t}$:

$$E_t\Delta\hat{Q}_{n,m,t+1} = \pi_{n,t} - E_t\pi_{n,t+1} - \pi_{m,t} + E_t\pi_{m,t+1} \tag{77}$$

Rewriting (77):

$$E_t\hat{Q}_{n,m,t+1} - \hat{Q}_{n,m,t} = \pi_{n,t} - E_t\pi_{n,t+1} - \pi_{m,t} + E_t\pi_{m,t+1}$$
$$\hat{Q}_{n,m,t} = E_t\hat{Q}_{n,m,t+1} + (E_t\pi_{n,t+1} - \pi_{n,t}) - (E_t\pi_{m,t+1} - \pi_{m,t})$$

Forwarding the previous equation yields:

$$\hat{Q}_{n,m,t} = E_t\left[\sum_{j=0}^{\infty}(\pi_{n,t+j+1} - \pi_{n,t+j}) - (\pi_{m,t+j+1} - \pi_{m,t+j})\right]$$

since under steady state stability long-run real variables will return to zero; that is $\lim_{t\to\infty}\hat{Q}_{n,m,t} = 0$. Everything other than initial inflation appears twice so it cancels out:

$$\hat{Q}_{n,m,t} = \pi_{m,t} - \pi_{n,t} \tag{78}$$

Using the definition of the real exchange rate in (75):

$$\hat{Q}_{n,m,t} = \hat{P}_{m,t} + \hat{\mathcal{E}}_{m,n,t} - \hat{P}_{n,t} = \pi_{m,t} - \pi_{n,t} \tag{79}$$

$$\hat{P}_{m,t} + \hat{\mathcal{E}}_{m,n,t} - \hat{P}_{n,t} = (\hat{P}_{m,t} - \hat{P}_{m,t-1}) - (\hat{P}_{n,t} - \hat{P}_{n,t-1}) \tag{80}$$

$$\hat{\mathcal{E}}_{m,n,t} = \hat{P}_{n,t-1} - \hat{P}_{m,t-1} \tag{81}$$

Equations (78) and (78) pin down the nominal and real exchange rates under the assumption that $\phi_\pi^n = \phi_\pi^m \to 1$. Similar to the approach above, in our analytical work instead of fully taking the limit to $\phi_\pi \to 1$, we assume $\phi_\pi \approx 1$.

D.3 Setting Up the Method of Undetermined Coefficients

Recall that:

$$\hat{\boldsymbol{P}}_t^P = \tilde{\boldsymbol{\Psi}}\left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\left(\hat{\boldsymbol{P}}_t^C + \hat{\boldsymbol{C}}_t\right) + [\boldsymbol{\Omega} \odot \hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} + [\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1}\right) + \boldsymbol{\beta}\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P\right]$$

Note that $\hat{\boldsymbol{P}}_t^C + \hat{\boldsymbol{C}}_t = \hat{\boldsymbol{P}}_t^C - \boldsymbol{\pi}_t = \hat{\boldsymbol{P}}_{t-1}^C$. Therefore we can write the equation of motion for the price indices as:

$$\hat{\boldsymbol{P}}_t^P = \tilde{\boldsymbol{\Psi}}\left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda}\boldsymbol{\alpha}\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{\Lambda}[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} + \boldsymbol{\Lambda}[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1} + \boldsymbol{\beta}\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P\right] \tag{82}$$

$$\hat{\boldsymbol{P}}_t^C = \boldsymbol{\Xi} \cdot \hat{\boldsymbol{P}}_t^P + [\boldsymbol{\Xi} \odot \hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} + [\boldsymbol{\Xi} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1} \tag{83}$$

Using Lemmas 1 and 2, and using Equation 81 above, we can write:

$$\begin{aligned}
[\boldsymbol{\Omega} \odot \hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} &= (\boldsymbol{I} - \boldsymbol{\alpha} - \boldsymbol{\Omega})\hat{\boldsymbol{P}}_{t-1}^C \\
[\boldsymbol{\Xi} \odot \hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} &= (\boldsymbol{I} - \boldsymbol{\Xi})\hat{\boldsymbol{P}}_{t-1}^C \\
\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t &= \breve{\boldsymbol{\Omega}}\breve{\boldsymbol{\tau}}_t \\
\boldsymbol{\Xi} \odot \hat{\boldsymbol{\tau}}_t &= \breve{\boldsymbol{\Xi}}\breve{\boldsymbol{\tau}}_t
\end{aligned}$$

Then we can write:

$$\hat{\boldsymbol{P}}_t^P = \tilde{\boldsymbol{\Psi}}\left[\hat{\boldsymbol{P}}_{t-1}^P + \underbrace{\boldsymbol{\Lambda}(\boldsymbol{I} - \boldsymbol{\Omega})}_{\boldsymbol{A}}\hat{\boldsymbol{P}}_{t-1}^C + \underbrace{\boldsymbol{\Lambda}\breve{\boldsymbol{\Omega}}}_{\boldsymbol{B}}\breve{\boldsymbol{\tau}}_t + \boldsymbol{\beta}\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P\right] \tag{84}$$

$$\hat{\boldsymbol{P}}_t^C = \boldsymbol{\Xi} \cdot \hat{\boldsymbol{P}}_t^P + \underbrace{(\boldsymbol{I} - \boldsymbol{\Xi})}_{\boldsymbol{D}}\hat{\boldsymbol{P}}_{t-1}^C + \breve{\boldsymbol{\Xi}}\breve{\boldsymbol{\tau}}_t \tag{85}$$

That is we have:

$$\begin{aligned}
\hat{\boldsymbol{P}}_t^P &= \tilde{\boldsymbol{\Psi}}\boldsymbol{P}_{t-1}^P + \tilde{\boldsymbol{\Psi}}\boldsymbol{A}\hat{\boldsymbol{P}}_{t-1}^C + \tilde{\boldsymbol{\Psi}}\boldsymbol{B}\tau_t + \tilde{\boldsymbol{\Psi}}\boldsymbol{\beta}(\mathbb{E}_t\boldsymbol{P}_{t+1}^P) \\
\hat{\boldsymbol{P}}_t^C &= \boldsymbol{\Xi}\boldsymbol{P}_t^P + \boldsymbol{D}\hat{\boldsymbol{P}}_{t-1}^C + \breve{\boldsymbol{\Xi}}\breve{\boldsymbol{\tau}}_t
\end{aligned}$$

We will now diagonalize $\tilde{\boldsymbol{\Psi}} = \boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}$. We then define:

$$\begin{aligned}
\breve{\boldsymbol{P}}_t^P &= \boldsymbol{Q}^{-1}\hat{\boldsymbol{P}}_t^P \\
\breve{\boldsymbol{A}} &= \boldsymbol{Q}^{-1}\boldsymbol{A} \\
\breve{\boldsymbol{B}} &= \boldsymbol{Q}^{-1}\boldsymbol{B}
\end{aligned}$$

$$\breve{\boldsymbol{\beta}} = \boldsymbol{Q}^{-1}\boldsymbol{\beta}\boldsymbol{Q}$$

So now the system is

$$\begin{aligned}
\breve{\boldsymbol{P}}_t^P &= \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{P}}_{t-1}^P + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{A}}\hat{\boldsymbol{P}}_{t-1}^C + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{B}}\breve{\boldsymbol{\tau}}_t + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}(\mathbb{E}_t\breve{\boldsymbol{P}}_{t+1}^P) \\
\hat{\boldsymbol{P}}_t^C &= \boldsymbol{\Xi}\boldsymbol{Q}\breve{\boldsymbol{P}}_t^P + \boldsymbol{D}\hat{\boldsymbol{P}}_{t-1}^C + \breve{\boldsymbol{\Xi}}\breve{\boldsymbol{\tau}}_t
\end{aligned}$$

Let us now postulate:

$$\begin{aligned}
\breve{\boldsymbol{P}}_t^P &= \underbrace{\boldsymbol{C}_1}_{NJ\times NJ}\breve{\boldsymbol{P}}_{t-1}^P + \underbrace{\boldsymbol{C}_2}_{NJ\times NJ}\hat{\boldsymbol{P}}_{t-1}^C + \underbrace{\boldsymbol{C}_3}_{NJ\times N^2J}\breve{\boldsymbol{\tau}}_t \\
\hat{\boldsymbol{P}}_t^C &= \underbrace{\boldsymbol{C}_4}_{NJ\times NJ}\boldsymbol{Q}\breve{\boldsymbol{P}}_{t-1}^P + \underbrace{\boldsymbol{C}_5}_{NJ\times NJ}\hat{\boldsymbol{P}}_{t-1}^C + \underbrace{\boldsymbol{C}_6}_{NJ\times N^2J}\breve{\boldsymbol{\tau}}_t
\end{aligned}$$

Iterating the first equation forward and taking expectation at time t, under the assumption that the tariff is a one-time shock:

$$\begin{aligned}
E_t\breve{\boldsymbol{P}}_{t+1}^P &= \boldsymbol{C}_1\breve{\boldsymbol{P}}_t^P + \boldsymbol{C}_2\hat{\boldsymbol{P}}_t^C \\
&= \boldsymbol{C}_1\left(\boldsymbol{C}_1\breve{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_2\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{C}_3\breve{\boldsymbol{\tau}}_t\right) + \boldsymbol{C}_2\left(\boldsymbol{C}_4\boldsymbol{Q}\breve{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_5\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{C}_6\breve{\boldsymbol{\tau}}_t\right)
\end{aligned}$$

Plugging these into the two original equations:

$$\begin{aligned}
\left(\boldsymbol{C}_1\breve{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_2\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{C}_3\breve{\boldsymbol{\tau}}_t\right) &= \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{P}}_{t-1}^P + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{A}}\hat{\boldsymbol{P}}_{t-1}^C + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{B}}\breve{\boldsymbol{\tau}}_t \\
&\quad + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\Bigg(\boldsymbol{C}_1\left(\boldsymbol{C}_1\breve{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_2\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{C}_3\breve{\boldsymbol{\tau}}_t\right) \\
&\quad + \boldsymbol{C}_2\left(\boldsymbol{C}_4\boldsymbol{Q}\breve{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_5\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{C}_6\breve{\boldsymbol{\tau}}_t\right)\Bigg) \\
\boldsymbol{C}_4\boldsymbol{Q}\breve{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_5\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{C}_6\breve{\boldsymbol{\tau}}_t &= \boldsymbol{\Xi}\boldsymbol{Q}\left(\boldsymbol{C}_1\breve{\boldsymbol{P}}_{t-1}^P + \boldsymbol{C}_2\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{C}_3\breve{\boldsymbol{\tau}}_t\right) + \boldsymbol{D}\hat{\boldsymbol{P}}_{t-1}^C + \breve{\boldsymbol{\Xi}}\tau_t
\end{aligned}$$

Expanding and grouping terms:

$$\begin{aligned}
&\left(\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{C}_4\boldsymbol{Q}\right)\breve{\boldsymbol{P}}_{t-1}^P + \left(\boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{A}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{C}_5\right)\hat{\boldsymbol{P}}_{t-1}^C \\
&+ \left(\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{B}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{C}_6\right)\breve{\boldsymbol{\tau}}_t = 0
\end{aligned}$$

And:

$$\left(\boldsymbol{C}_4\boldsymbol{Q} - \boldsymbol{\Xi}\boldsymbol{Q}\boldsymbol{C}_1\right)\breve{\boldsymbol{P}}_{t-1}^P + \left(\boldsymbol{C}_5 - \boldsymbol{\Xi}\boldsymbol{Q}\boldsymbol{C}_2 - \boldsymbol{D}\right)\hat{\boldsymbol{P}}_{t-1}^C + \left(\boldsymbol{C}_6 - \boldsymbol{\Xi}\boldsymbol{Q}\boldsymbol{C}_3 - \breve{\boldsymbol{\Xi}}\right)\breve{\boldsymbol{\tau}}_t = 0$$

This yields a system of 6 (matrix) equations and 6 unknowns:

$$\begin{aligned}
&\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{C}_4\boldsymbol{Q} = 0\\
&\boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{A}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{C}_5 = 0\\
&\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{B}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{C}_6 = 0\\
&\boldsymbol{C}_4\boldsymbol{Q} - \boldsymbol{\Xi}\boldsymbol{Q}\boldsymbol{C}_1 = 0\\
&\boldsymbol{C}_5 - \boldsymbol{\Xi}\boldsymbol{Q}\boldsymbol{C}_2 - \boldsymbol{D} = 0\\
&\boldsymbol{C}_6 - \boldsymbol{\Xi}\boldsymbol{Q}\boldsymbol{C}_3 - \breve{\boldsymbol{\Xi}} = 0
\end{aligned}$$

<u>Dependent Blocks</u>

$$\begin{aligned}
\mathbf{C}_4 &= \boldsymbol{\Xi}\boldsymbol{Q}\boldsymbol{C}_1\boldsymbol{Q}^{-1},\\
\mathbf{C}_5 &= \boldsymbol{D} + \boldsymbol{\Xi}\boldsymbol{Q}\boldsymbol{C}_2,\\
\mathbf{C}_6 &= \boldsymbol{\Xi}\boldsymbol{Q}\boldsymbol{C}_3 + \breve{\boldsymbol{\Xi}},
\end{aligned}$$

<u>Core Fixed-Point Equations</u>

$$\begin{aligned}
&\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_1 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{\Xi}\boldsymbol{Q}\boldsymbol{C}_1 = 0\\
&\boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{A}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_2 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{D} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{\Xi}\boldsymbol{Q}\boldsymbol{C}_2 = 0\\
&\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{B}} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_1\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{\Xi}\boldsymbol{Q}\boldsymbol{C}_3 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\breve{\boldsymbol{\Xi}} = 0
\end{aligned}$$

After multiplying on the left by $\breve{\boldsymbol{\Psi}}^{-}1$, the first equation can be rewritten as:

$$\begin{aligned}
\underbrace{\left(\breve{\boldsymbol{\Psi}}^{-1} - \breve{\boldsymbol{\beta}}\mathbf{C}_1 - \breve{\boldsymbol{\beta}}\mathbf{C}_2\boldsymbol{\Xi}\boldsymbol{Q}\right)}_{=\mathbf{C}_1^{-1}}\mathbf{C}_1 &= \boldsymbol{I}\\
\breve{\boldsymbol{\Psi}}\mathbf{C}_1^{-1} &= \boldsymbol{I} - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\mathbf{C}_1 - \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\mathbf{C}_2\boldsymbol{\Xi}\boldsymbol{Q}
\end{aligned}$$

Plugging this expression into the second and third equations gives us:

$$\begin{aligned}
&\breve{\boldsymbol{\Psi}}\breve{\boldsymbol{A}} + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{D} = \breve{\boldsymbol{\Psi}}\boldsymbol{C}_1^{-1}\boldsymbol{C}_2 \Rightarrow \boldsymbol{C}_2 = \boldsymbol{C}_1(\breve{\boldsymbol{A}} + \breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{D})\\
&\breve{\boldsymbol{\Psi}}\breve{\boldsymbol{B}} + \breve{\boldsymbol{\Psi}}\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\breve{\boldsymbol{\Xi}} = \breve{\boldsymbol{\Psi}}\boldsymbol{C}_1^{-1}\boldsymbol{C}_3 \Rightarrow \boldsymbol{C}_3 = \boldsymbol{C}_1(\breve{\boldsymbol{B}} + \breve{\boldsymbol{\beta}}\boldsymbol{C}_2\breve{\boldsymbol{\Xi}})
\end{aligned}$$

Hence, $\boldsymbol{C}_3$ can be written as a function of $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$. So we need to solve for these two matrices.

We can rewrite the first equation:

$$\breve{\boldsymbol{\beta}}\mathbf{C}_1^2 - (\breve{\boldsymbol{\Psi}}^{-1} + \breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{\Xi}\boldsymbol{Q})\mathbf{C}_1 + \boldsymbol{I} = \mathbf{0}$$

This expression, along with the expression for $\boldsymbol{C}_2$ can be numerically solved. Here we will make two simplifying assumptions to arrive at an analytical expression. First, we assume all discount factors are the same for the countries, i.e., $\beta_n = \beta \quad \forall n$, $\tilde{\boldsymbol{\beta}}$ becomes $\beta\boldsymbol{I}$. Second, we will ignore the term $\breve{\boldsymbol{\beta}}\boldsymbol{C}_2\boldsymbol{\Xi}\boldsymbol{Q}$ since this term is relatively small number numerically. With these simplifying assumptuons, we can now solve for $\mathbf{C}_1$ with the quadratic formula. We wish to solve for the diagonal matrix $\mathbf{C}_1$ in

$$\beta\,\mathbf{C}_1^2 - \breve{\boldsymbol{\Psi}}^{-1}\mathbf{C}_1 + \mathbf{I} = \mathbf{0},$$

assuming

$$\mathbf{C}_1 = \operatorname{diag}(c_1, c_2, \ldots, c_n) \quad \text{and} \quad \breve{\boldsymbol{\Psi}} = \operatorname{diag}(\psi_1, \psi_2, \ldots, \psi_n).$$

For each i, the i-th diagonal element satisfies

$$\beta\,c_i^2 - \frac{1}{\psi_i}c_i + 1 = 0.$$

Dividing by β yields

$$c_i^2 - \frac{1}{\beta\psi_i}\,c_i + \frac{1}{\beta} = 0.$$

Applying the quadratic formula gives

$$c_i = \frac{\frac{1}{\psi_i} \pm \sqrt{\frac{1}{\psi_i^2} - 4\beta}}{2\beta}$$

With $\mathbf{C}_1$ is close to $\overline{\rho}\mathbf{I}$, where $\overline{\rho}$ is the average of the elements in the diagonal, we can now solve for $\mathbf{C}_2$

$$\mathbf{C}_2 = \overline{\rho}\breve{\boldsymbol{A}}(\mathbf{I} - \beta\overline{\rho}\mathbf{D})^{-1}$$

Finally, $\mathbf{C}_3$ is given by:

$$\boldsymbol{C}_3 = \overline{\rho}\breve{\boldsymbol{B}} + \beta\overline{\rho}\breve{\boldsymbol{A}}(\mathbf{I} - \beta\overline{\rho}\mathbf{D})^{-1}\breve{\boldsymbol{\Xi}}$$

With these we can now return to $\boldsymbol{C}_6$, our object of interest which captures the impact of tariffs on consumer price inflation.

$$
\begin{aligned}
\mathbf{C}_6 &= \boldsymbol{\Xi}\boldsymbol{Q}\mathbf{C}_3 + \breve{\boldsymbol{\Xi}} \\
&= \boldsymbol{\Xi}\boldsymbol{Q}\mathbf{C}_1\boldsymbol{Q}^{-1}\left(\overline{\rho}\boldsymbol{\Lambda}\breve{\boldsymbol{\Omega}} + \beta\overline{\rho}\boldsymbol{A}(\mathbf{I}-\beta\overline{\rho}\mathbf{D})^{-1}\breve{\boldsymbol{\Xi}}\right) + \breve{\boldsymbol{\Xi}}
\end{aligned}
$$

where we used $\boldsymbol{B} = \boldsymbol{\Lambda}\breve{\boldsymbol{\Omega}}$.

Now let's assume that the countries increase their tariffs with the same amount $\hat{\tau}_{n,mi,t} = \hat{\tau}$ and $\hat{\tau}_{n,ni,t} = 0, \quad \forall n, ni, mi$. The second equation specifies that there are no tariffs domestically. With these assumptions,

$$
\begin{aligned}
\breve{\boldsymbol{\Xi}}\breve{\boldsymbol{\tau}}_t &= \tilde{\boldsymbol{\Xi}}^F\hat{\tau} \\
\breve{\boldsymbol{\Omega}}\breve{\boldsymbol{\tau}}_t &= \tilde{\boldsymbol{\Omega}}^F\hat{\tau}
\end{aligned}
$$

where $\tilde{\boldsymbol{\Xi}}^F$ and $\tilde{\boldsymbol{\Omega}}^F$ are $NJ \times 1$ dimensional vectors that represent the foreign weight in the final consumption and the inputs, respectively. Hence:

$$
\frac{\partial \hat{\boldsymbol{P}}_t^C}{\partial \hat{\tau}_t} = \boldsymbol{\Xi}\boldsymbol{Q}\mathbf{C}_1\boldsymbol{Q}^{-1}\left(\overline{\rho}\boldsymbol{\Lambda}\tilde{\boldsymbol{\Omega}}^F + \beta\overline{\rho}\boldsymbol{A}(\mathbf{I}-\beta\overline{\rho}\mathbf{D})^{-1}\tilde{\boldsymbol{\Xi}}^F\right) + \tilde{\boldsymbol{\Xi}}^F
$$

where $\boldsymbol{Q}$ comes from the diagonalization of the stickiness-adjusted Leontief inverse: $\tilde{\boldsymbol{\Psi}} = \boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}$. Let us call this $\boldsymbol{Q}\mathbf{C}_1\boldsymbol{Q}^{-1} = \boldsymbol{\Psi}^{NKOE}$, indicating that this is now the New Keynesian Open Economy Leontief inverse (taking the stickiness adjusted Leontief inverse to NKOE setting with expectations). Let us now define loadings:

$$
\begin{aligned}
\boldsymbol{A} &= \boldsymbol{\Lambda}(\boldsymbol{L}_C^P + \boldsymbol{L}_{\mathcal{E}}^P) \\
\boldsymbol{B} &= \boldsymbol{\Lambda}\boldsymbol{L}_{\tau}^P \\
\overline{\rho}(\mathbf{I}-\beta\overline{\rho}\boldsymbol{D})^{-1} &= \boldsymbol{L}_{\mathcal{E}}^C \\
\boldsymbol{F} &= \boldsymbol{L}_{\tau}^C
\end{aligned}
$$

Then:

$$
\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \boldsymbol{\beta}\boldsymbol{\Psi}^{NKOE}\boldsymbol{\Lambda}(\boldsymbol{L}_{\tau}^P + \beta(\boldsymbol{L}_C^P + \boldsymbol{L}_{\mathcal{E}}^P)\boldsymbol{L}_{\mathcal{E}}^C\boldsymbol{L}_{\tau}^C) + \boldsymbol{L}_{\tau}^C
$$

D.4 Generalizing the Result: Two Country Case

If $\phi_\pi \to 1$ is not the case, in the general case only the loadings change. This is because $\hat{W} - \hat{P}_t^C = -\hat{P}_t^C + \phi_\pi \hat{P}_t^C$ and the exchange rate is more generally

$$\hat{E}_t = \overline{E} + \phi_\pi \hat{P}_{t-1}^C - \phi_\pi^* \hat{P}_{t-1}^{*C}$$

We know both from numerical simulations and similar models that the $\overline{E}$ will be a function of the real debt position. Since it is linearly separable and the quantitative impact is small when the elasticities of substitution are small (i.e. below 1 indicating goods are complements on the production side), we will momentarily ignore it in the following section.

That is in vector form, in the two-country case we have $\hat{\mathbf{W}}_t = \boldsymbol{\Phi}\hat{\boldsymbol{P}}_{t-1}^C$ and $\hat{\mathcal{E}}_t \approx \tilde{\boldsymbol{\Phi}}\hat{\boldsymbol{P}}_{t-1}^C$ where $\tilde{\boldsymbol{\Phi}} = \begin{bmatrix} 1 & -1 \end{bmatrix} \boldsymbol{\Phi}$. With $\begin{bmatrix} 1 & -1 \end{bmatrix}$ already defined within the loading, this means all that changes is:

$$\boldsymbol{A} = \boldsymbol{\Lambda}(\boldsymbol{L}_C^P + \boldsymbol{L}_{\mathcal{E}}^P)\boldsymbol{\Phi}$$

Then:

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \boldsymbol{\beta}\boldsymbol{\Psi}^{NKOE}\boldsymbol{\Lambda}(\boldsymbol{L}_\tau^P + \beta(\boldsymbol{L}_C^P + \boldsymbol{L}_{\mathcal{E}}^P)\boldsymbol{\Phi}\boldsymbol{L}_{\mathcal{E}}^C\boldsymbol{L}_\tau^C) + \boldsymbol{L}_\tau^C$$

D.4.1 Impact of Policy

$$\hat{\boldsymbol{P}}_t^P = \underbrace{(\boldsymbol{I}(1+\beta) + \boldsymbol{\Lambda}(\boldsymbol{I} - \boldsymbol{\Omega}))^{-1}}_{\tilde{\boldsymbol{\Psi}}} \left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda}\left(\boldsymbol{\alpha} \underbrace{\left(\hat{\boldsymbol{P}}_t^C + \sigma\hat{\boldsymbol{C}}_t\right)}_{\hat{\boldsymbol{W}}_t} + [\boldsymbol{\Omega} \odot \hat{\boldsymbol{\mathcal{E}}}_t]\mathbf{1} + [\boldsymbol{\Omega} \odot \hat{\boldsymbol{\tau}}_t]\mathbf{1} \right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P \right]$$

where $\tilde{\boldsymbol{\Psi}}$ is a stickiness-adjusted Leontief Inverse. Let us plug in our approximation of the Euler equation:

$$\hat{\boldsymbol{C}}_t = -\boldsymbol{\Phi}(\boldsymbol{P}_t^C - \boldsymbol{P}_{t-1}^C)$$

which implies under $\sigma = 1$:

$$\hat{\boldsymbol{W}}_t = \hat{\mathbf{P}}_t^C + \hat{\boldsymbol{C}}_t = (\mathbf{I} - \boldsymbol{\Phi})\hat{\mathbf{P}}_t^C - \boldsymbol{\Phi}\boldsymbol{P}_{t-1}^C$$

We also have in vector form, in the two-country case $\hat{\mathbf{W}}_t = \boldsymbol{\Phi}\hat{\boldsymbol{P}}_{t-1}^C$. Plugging this into the NKPC:

$$(\boldsymbol{I}(1+\beta)+\boldsymbol{\Lambda}(\boldsymbol{I}-\boldsymbol{\Omega}))\hat{\boldsymbol{P}}_t^P = \left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\left((\mathbf{I}-\boldsymbol{\Phi})\hat{\mathbf{P}}_t^C - \boldsymbol{\Phi}\boldsymbol{P}_{t-1}^C\right) + \mathbf{L}_{\hat{\mathcal{E}}}^P\hat{\mathcal{E}}_t + \mathbf{L}_\tau^P\tau_t\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P\right]$$

Next we substitute out consumer prices, using $\hat{\boldsymbol{P}}_t^C = \boldsymbol{\Xi}\hat{\boldsymbol{P}}_t^P + \boldsymbol{D}\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{L}_\tau^C\tau_t$ and the exchange rate given $\hat{\mathcal{E}}_t \approx \tilde{\boldsymbol{\Phi}}\hat{\boldsymbol{P}}_{t-1}^C$ where $\tilde{\boldsymbol{\Phi}} = \begin{bmatrix}1 & -1\end{bmatrix}\boldsymbol{\Phi}$. With $\begin{bmatrix}1 & -1\end{bmatrix}$ already defined within the loading:

$$(\boldsymbol{I}(1+\beta)+\boldsymbol{\Lambda}(\boldsymbol{I}-\boldsymbol{\Omega}))\hat{\boldsymbol{P}}_t^P =$$
$$\left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda}\left(\boldsymbol{L}_C^P\left((\mathbf{I}-\boldsymbol{\Phi})\left(\boldsymbol{\Xi}\hat{\boldsymbol{P}}_t^P + \boldsymbol{D}\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{L}_\tau^C\tau_t\right) - \boldsymbol{\Phi}\boldsymbol{P}_{t-1}^C\right) + \mathbf{L}_{\hat{\mathcal{E}}}^P\hat{\boldsymbol{P}}_{t-1}^C + \mathbf{L}_\tau^P\tau_t\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P\right]$$

Grouping terms and rearranging:

$$\hat{\boldsymbol{P}}_t^P = \underbrace{\left[\boldsymbol{I}(1+\beta)+\boldsymbol{\Lambda}\big[\boldsymbol{I}-\boldsymbol{\Omega}+\boldsymbol{L}_C^P(\boldsymbol{\Phi}-\boldsymbol{I})\Xi\big]\right]^{-1}}_{\tilde{\boldsymbol{\Psi}}_\phi}$$
$$\left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda}\left[\big[\boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{D} + \mathbf{L}_{\hat{\mathcal{E}}}^P - \boldsymbol{L}_C^P\big]\boldsymbol{\Phi}\boldsymbol{P}_{t-1}^C + \big[\boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{L}_\tau^C + \mathbf{L}_\tau^P\big]\tau_t\right] + \beta\mathbb{E}t\hat{\boldsymbol{P}}_{t+1}^P\right]$$

Going back to earlier solution we have:

$$\frac{\partial\hat{\boldsymbol{P}}_t^C}{\partial\hat{\tau}_t} = \boldsymbol{\Xi}\boldsymbol{Q}\mathbf{C}_1\boldsymbol{Q}^{-1}\left(\overline{\rho}\boldsymbol{\Lambda}\tilde{\boldsymbol{\Omega}}^F + \beta\overline{\rho}\boldsymbol{A}(\mathbf{I}-\beta\overline{\rho}\mathbf{D})^{-1}\tilde{\boldsymbol{\Xi}}^F\right) + \tilde{\boldsymbol{\Xi}}^F$$

Or alternatively:

$$\frac{\partial\hat{\boldsymbol{P}}_t^C}{\partial\tau_t} = \boldsymbol{\Xi}\boldsymbol{Q}\mathbf{C}_1\boldsymbol{Q}^{-1}\left(\boldsymbol{B} + \beta\overline{\rho}\boldsymbol{A}(\mathbf{I}-\beta\overline{\rho}\mathbf{D})^{-1}\boldsymbol{F}\right) + \boldsymbol{F}$$

Let us now define loadings (keeping in mind that the cross term $\boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{D} \approx 0$ due to home bias, so for narrative simplicity we'll omit it in the expression below):

$$\begin{aligned}
\boldsymbol{A} &= \big[\boldsymbol{L}_C^P + \boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{D} + \mathbf{L}_{\hat{\mathcal{E}}}^P\big]\boldsymbol{\Phi} \to \boldsymbol{\Lambda}(\boldsymbol{L}_C^P + \boldsymbol{L}_{\mathcal{E}}^P)\boldsymbol{\Phi} \\
\boldsymbol{B} &= \big[\boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{L}_\tau^C + \mathbf{L}_\tau^P\big] \\
\overline{\rho}(\mathbf{I}-\beta\overline{\rho}\boldsymbol{D})^{-1} &= \boldsymbol{L}_{\mathcal{E}}^C \\
\boldsymbol{F} &= \boldsymbol{L}_\tau^C
\end{aligned}$$

Then:

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \boldsymbol{\Xi}\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda}\left[\boldsymbol{L}_\tau^P + \left(\boldsymbol{L}_C^P(\mathbf{I} - \boldsymbol{\Phi}) + \beta(\boldsymbol{L}_C^P + \boldsymbol{L}_\mathcal{E}^P)\boldsymbol{\Phi}\boldsymbol{L}_\mathcal{E}^C\right)\boldsymbol{L}_\tau^C\right] + \boldsymbol{L}_\tau^C$$

D.5 Examples

D.5.1 Case 1: N=1,J=1, standard NK model

We can begin by comparing how the model and its solution to the three-equation canonical New Keynesian model recopied below. For simplicity, let us have demand shocks given by ϵ_t^d and supply (cost-push) shocks given by μ_t:

$$\begin{aligned}\sigma(\mathbb{E}_t\hat{Y}_{t+1} - \hat{Y}_t) &= \hat{i}_t - \mathbb{E}_t\pi_{t+1} + \epsilon^d \\ \pi_t &= \kappa\hat{Y}_t + \mu_t + \beta\mathbb{E}_t\pi_{t+1}^P \\ \hat{i}_t &= \phi_\pi\pi_t\end{aligned}$$

The standard solution in this model for π_t, when the shocks in question are one-time shocks, reads as follows:

$$\pi_t = \frac{\sigma}{\kappa\phi_\pi + \sigma}\mu_t + \frac{\kappa}{\kappa\phi_\pi + \sigma}\epsilon_t^d \tag{86}$$

We can reduce our model to the scalar case, by setting $N = 1$ and $J = 1$ to compare our solution to the standard one. Relative to the general case with N countries and J industries, the exchange rate drops out and τ_t on the production side is isomorphic to a cost-push shock. Additionally, lagged prices disappear. In a closed economy there would not be tariffs. However, to see the analogy and the intuition here we can treat $\epsilon_t^d = L_\tau^C\tau_t$ as a demand shock as a wedge between producer prices and consumer prices would be isomorphic to one (i.e. the loading in this analogy would be different as we show below). $\kappa = \Lambda L_C^P$ would be the slope of the NKPC and let $\mu_t = \Lambda L_\tau^P\tau_t$ be a cost-push shock. Written with the notation we developed, with the Taylor rule plugged in, and keeping $\sigma = 1, \psi = 0$ we would have:

$$\begin{aligned}\mathbb{E}_t\hat{Y}_{t+1} - \hat{Y}_t &= \underbrace{\phi_\pi\pi_t}_{\hat{i}_t} - \mathbb{E}_t\pi_{t+1} + \underbrace{L_\tau^C\tau_t}_{\epsilon_t^d} \\ \pi_t &= \underbrace{\Lambda L_C^P}_{\kappa}\hat{Y}_t + \underbrace{\Lambda L_\tau^P\tau_t}_{\mu_t} + \beta\mathbb{E}_t\pi_{t+1}^P\end{aligned}$$

Plugging in the parameters into the standard solution in (86) we find:

$$\pi_t = \frac{\Lambda}{1 + \phi_\pi \Lambda L_C^P} \left[L_\tau^P + L_C^P L_\tau^C \right] \tau_t \tag{87}$$

After performing an adjustment for the fact that our model's solution was derived in a setup with lags, this would be the same as the solution in (44).

D.5.2 Case 2: N=2, J=1, no intermediate inputs

This set up is similar to the one solved by Monacelli (2025). Here, I-O matrix is a matrix of zeros, i.e., $\mathbf{\Omega} = \mathbf{0}$. Then:

$$(\boldsymbol{I}(1+\beta) + \mathbf{\Lambda})\hat{\boldsymbol{P}}_t^P = \left[\hat{\boldsymbol{P}}_{t-1}^P + \mathbf{\Lambda}\left((\mathbf{I} - \mathbf{\Phi})\hat{\mathbf{P}}_t^C - \mathbf{\Phi}\boldsymbol{P}_{t-1}^C \right) + \beta \mathbb{E}_t \hat{\boldsymbol{P}}_{t+1}^P \right]$$

Next we substitute out consumer prices, using

$$\hat{\boldsymbol{P}}_t^C = \mathbf{\Xi}\hat{\boldsymbol{P}}_t^P + \boldsymbol{D}\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{L}_\tau^C \tau_t$$

we arrive at:

$$\begin{aligned} &(\boldsymbol{I}(1+\beta) + \mathbf{\Lambda})\hat{\boldsymbol{P}}_t^P = \\ &\left[\hat{\boldsymbol{P}}_{t-1}^P + \mathbf{\Lambda}\left((\mathbf{I} - \mathbf{\Phi})\left(\mathbf{\Xi}\hat{\boldsymbol{P}}_t^P + \boldsymbol{D}\hat{\boldsymbol{P}}_{t-1}^C + \boldsymbol{L}_\tau^C \tau_t \right) - \mathbf{\Phi}\boldsymbol{P}_{t-1}^C \right) + \beta \mathbb{E}_t \hat{\boldsymbol{P}}_{t+1}^P \right] \end{aligned}$$

Grouping terms and rearranging:

$$\begin{aligned} \hat{\boldsymbol{P}}_t^P = &\underbrace{\left[\boldsymbol{I}(1+\beta) + \mathbf{\Lambda}\big[\mathbf{I} + (\mathbf{\Phi} - \boldsymbol{I})\mathbf{\Xi} \big] \right]^{-1}}_{\tilde{\mathbf{\Psi}}_{\boldsymbol{\phi}}} \\ &\left[\hat{\boldsymbol{P}}_{t-1}^P + \mathbf{\Lambda}\left[\big[(\mathbf{I} - \mathbf{\Phi})(\mathbf{I} - \mathbf{\Xi}) \big] \mathbf{\Phi}\boldsymbol{P}_{t-1}^C + (\mathbf{I} - \mathbf{\Phi})\boldsymbol{L}_\tau^C \tau_t + \beta \mathbb{E}_t \hat{\boldsymbol{P}}_{t+1}^P \right] \right. \end{aligned}$$

Let's assume the matrices are defined as:

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \qquad \mathbf{\Phi} = \begin{bmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{bmatrix}, \qquad \mathbf{\Xi} = \begin{bmatrix} \xi_1 & (1-\xi_1) \\ (1-\xi_2) & \xi_2 \end{bmatrix}$$

with ξ_1 and ξ_2 capturing the domestic consumption bias of home and foreign, respectively. Then $\tilde{\Psi}_\phi$ is given by:

$$\tilde{\boldsymbol{\Psi}}_\phi = \Bigg[\boldsymbol{I}(1+\beta) + \boldsymbol{\Lambda}\big[\mathbf{I} + (\boldsymbol{\Phi} - \boldsymbol{I})\boldsymbol{\Xi}\big]\Bigg]^{-1} = \frac{1}{\Delta}\begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

where

$$\begin{aligned}
&A = 1 + \beta + \lambda_1(1 + \xi_1(\phi_1 - 1)), \quad B = \lambda_1(1 - \xi_1)(\phi_1 - 1) \\
&C = \lambda_2(1 - \xi_2)(\phi_2 - 1), \quad D = 1 + \beta + \lambda_2(1 + \xi_2(\phi_2 - 1)) \\
&\Delta = (1 + \beta + \lambda_1(1 + \xi_1(\phi_1 - 1)))(1 + \beta + \lambda_2(1 + \xi_2(\phi_2 - 1))) - \lambda_1\lambda_2(1 - \xi_1)(\phi_1 - 1)(1 - \xi_2)(\phi_2 - 1)
\end{aligned}$$

Let's assume symmetric countries with $\phi_1 = \phi_2 = \phi$, $\lambda_1 = \lambda_2 = \lambda$ and $\xi_1 = \xi_2 = \xi$. Then the expression simplifies to:

$$\tilde{\boldsymbol{\Psi}}_\phi = \frac{1}{\Delta}\begin{bmatrix} 1 + \beta + \lambda(1 + \xi(\phi - 1)) & -\lambda(1 - \xi)(\phi - 1) \\ -\lambda(1 - \xi)(\phi - 1) & 1 + \beta + \lambda(1 + \xi(\phi - 1)) \end{bmatrix}$$

where

$$\Delta = (1 + \beta)^2 + 2(1 + \beta)\lambda(1 + \xi(\phi - 1)) + 4\lambda^2\xi(\phi - 1)$$

If we do the eigendecomposition of $\tilde{\boldsymbol{\Psi}}_\phi$ such that $\tilde{\boldsymbol{\Psi}}_\phi = \boldsymbol{Q}\breve{\boldsymbol{\Psi}}\boldsymbol{Q}^{-1}$, then:

$$\breve{\boldsymbol{\Psi}} = \begin{bmatrix} \tilde{\psi}_1 & 0 \\ 0 & \tilde{\psi}_2 \end{bmatrix}, \qquad \boldsymbol{Q} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \qquad \boldsymbol{Q}^{-1} = \boldsymbol{Q}^T = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

where the eigenvalues $\tilde{\psi}_1$ and $\tilde{\psi}_2$ are given by:

$$\begin{aligned}
\tilde{\psi}_1 &= \frac{1 + \beta + \lambda(1 + \xi(\phi - 1)) - \lambda(1 - \xi)(\phi - 1)}{\Delta}, \\
\tilde{\psi}_2 &= \frac{1 + \beta + \lambda(1 + \xi(\phi - 1)) + \lambda(1 - \xi)(\phi - 1)}{\Delta}.
\end{aligned}$$

Now, we can solve for: $\beta\,\mathbf{C}_1^2 - \breve{\boldsymbol{\Psi}}^{-1}\mathbf{C}_1 + \mathbf{I} = \mathbf{0}$:

$$\mathbf{C}_1 = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}, \qquad c_1 = \frac{\frac{1}{\tilde{\psi}_1} \pm \sqrt{\left(\frac{1}{\tilde{\psi}_1}\right)^2 - 4\beta}}{2\beta}, \qquad c_2 = \frac{\frac{1}{\tilde{\psi}_2} \pm \sqrt{\left(\frac{1}{\tilde{\psi}_2}\right)^2 - 4\beta}}{2\beta}.$$

Then:

$$\tilde{\boldsymbol{\Psi}}_{\phi}^{NKOE} \approx \boldsymbol{Q}\mathbf{C}_1\boldsymbol{Q}^{-1} = \frac{1}{2}\begin{bmatrix} c_1+c_2 & c_1-c_2 \\ c_1-c_2 & c_1+c_2 \end{bmatrix}$$

Going back to earlier solution we have:

$$\frac{\partial \hat{\boldsymbol{P}}_t^C}{\partial \hat{\tau}_t} = \beta \boldsymbol{\Xi} \tilde{\boldsymbol{\Psi}}_{\phi}^{NKOE} \overline{\rho} \boldsymbol{\Lambda}(\mathbf{I} - \beta\overline{\rho}(\boldsymbol{I} - \boldsymbol{\Xi}))^{-1} \tilde{\boldsymbol{\Xi}}^F \mathbf{1} + \tilde{\boldsymbol{\Xi}}^F \mathbf{1}$$

where $\overline{\rho} = (c_1+c_2)/2$ and $\tilde{\boldsymbol{\Xi}}^F\mathbf{1} = [1-\xi, 1-\xi]^T$. Hence:

$$\frac{\partial \hat{\boldsymbol{P}}_t^C}{\partial \hat{\tau}_t} = \beta(1-\xi) \boldsymbol{\Xi} \tilde{\boldsymbol{\Psi}}_{\phi}^{NKOE} \overline{\rho} \boldsymbol{\Lambda}(\mathbf{I} - \beta\overline{\rho}(\boldsymbol{I} - \boldsymbol{\Xi}))^{-1} \mathbf{1} + (1-\xi)\mathbf{1},$$

where we resize $\mathbf{1}$ vector to $N \times 1$ dimensions.

E Relating the Balance of Payments to Prices

We can rewrite the BoP as follows:

$$\begin{aligned}
&\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{C}_{n,mj}(\hat{P}_{n,mj,t} + \hat{C}_{n,mj,t}) + \sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{X}_{ni,mj}(\hat{P}_{n,mj,t} + \hat{X}_{ni,mj,t}) \\
&+ \bar{\mathcal{E}}_n(1+\bar{i}_n^{US})\bar{B}_n^{US}\left(\hat{\mathcal{E}}_{n,t} + \hat{i}_{n,t-1}^{US} + \hat{B}_{n,t-1}^{US}\right) = \sum_i \bar{P}_{ni}\bar{Y}_{ni}(\hat{P}_{ni,t} + \hat{Y}_{ni,t}) + \bar{\mathcal{E}}_n\bar{B}_n^{US}(\hat{\mathcal{E}}_{n,t} + \hat{B}_{n,t}^{US}) \\
&\bar{\mathcal{E}}_n(1+\bar{i}_n^{US})\bar{B}_n^{US}\left(\hat{\mathcal{E}}_{n,t} + \hat{i}_{n,t-1}^{US} + \hat{B}_{n,t-1}^{US}\right) = \overline{NX}_n\widehat{NX}_{n,t} + \bar{\mathcal{E}}_n\bar{B}_n^{US}(\hat{\mathcal{E}}_{n,t} + \hat{B}_{n,t}^{US})
\end{aligned}$$

Redefining $\hat{V}_t$ as real dollar-denominated debt inclusive of interest payments and deflated by the local consumer price index: $\hat{V}_t = \frac{B_{n,t}^{US}(1+i_t)}{\hat{P}_t}$:

$$\bar{\mathcal{E}}_n\bar{V}_n^{US}\left(\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t-1}^{US} - \pi_t^C\right) = \overline{NX}_n\widehat{NX}_{n,t} + \frac{\bar{\mathcal{E}}_n\bar{V}_n^{US}}{1+\bar{i}^{US}}(\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t}^{US} - \hat{i}_t^{US})$$

WLOG $\bar{\mathcal{E}}_n = 1$. Also noting $(1+\bar{i}_n^{US}) = \beta^{-1}$, and $\overline{NX} = (1-\beta)\bar{V}_n^{US}$

$$\begin{aligned}
\bar{V}_n^{US}\left(\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t-1}^{US}\right) &= (1-\beta)\bar{V}_n^{US}\widehat{NX}_{n,t} + \beta\bar{V}_n^{US}(\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t}^{US} - \hat{i}_t^{US}) \\
\left(\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t-1}^{US}\right) &= (1-\beta)\widehat{NX}_{n,t} + \beta(\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t}^{US} - \hat{i}_t^{US}) \\
(1-\beta)\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t-1}^{US} &= (1-\beta)\widehat{NX}_{n,t} + \beta\hat{V}_{n,t}^{US} - \beta\hat{i}_t^{US} \\
\beta\hat{V}_{n,t}^{US} - \hat{V}_{n,t-1}^{US} &= (1-\beta)\hat{\mathcal{E}}_{n,t} - (1-\beta)\widehat{NX}_{n,t} + \beta\hat{i}_t^{US}
\end{aligned}$$

Using the market clearing condition and production function in vector notation, we can then express net exports as a function of prices. This yields the fifth equation in the five-equation representation.

$$\underbrace{\widehat{NX}_{n,t}}_{1\times 1} = \underbrace{\boldsymbol{S}_1}_{1\times NJ}\left(\underbrace{\hat{\mathbf{Y}}_t^{ni}}_{NJ\times 1} + \underbrace{\hat{\boldsymbol{P}}_t^{P}}_{NJ\times 1}\right) - \underbrace{\boldsymbol{S}_2}_{1\times N}\left(\underbrace{\hat{\boldsymbol{C}}_t}_{N\times 1} + \underbrace{\hat{\boldsymbol{P}}_t^{C}}_{N\times 1}\right) - \underbrace{\boldsymbol{S}_3}_{1\times NJNJ}\left(\underbrace{\hat{\boldsymbol{X}}_t}_{NJNJ\times 1} + \underbrace{\hat{\boldsymbol{P}}_t^{nimj}}_{NJNJ\times 1}\right) \tag{88}$$

where $\mathbf{S}$ denotes selector matrices. For example, $\mathbf{S}_1$ selects the country for whom net exports are calculated and additionally includes steady state ratios (e.g. $\frac{\overline{Y}^{ni}\overline{P}_{ni}^{P}}{\overline{NX}}$ if country is n and 0 if not).

Note vector of end user prices can be written as follows for firms and consumers:

$$\underbrace{\hat{\boldsymbol{P}}_t^{X}}_{NJNJ\times 1} = \left(\underbrace{\boldsymbol{S}_4}_{NJNJ\times NJ}\underbrace{\hat{\boldsymbol{P}}_t^{P}}_{NJ\times 1} + \underbrace{\boldsymbol{S}_5}_{NJNJ\times 1}\underbrace{\hat{\mathcal{E}}_t}_{1\times 1} + \underbrace{\boldsymbol{S}_6}_{NJNJ\times 1}\underbrace{\tau_t}_{1\times 1}\right) \tag{89}$$

$$\underbrace{\hat{\boldsymbol{P}}_t^{CX}}_{NNJ\times 1} = \left(\underbrace{\boldsymbol{S}_7}_{NNJ\times NJ}\underbrace{\hat{\boldsymbol{P}}_t^{P}}_{NJ\times 1} + \underbrace{\boldsymbol{S}_8}_{NNJ\times 1}\underbrace{\hat{\mathcal{E}}_t}_{1\times 1} + \underbrace{\boldsymbol{S}_9}_{NNJ\times 1}\underbrace{\tau_t}_{1\times 1}\right) \tag{90}$$

Market clearing conditions can be written as follows:

$$\underbrace{\hat{\mathbf{Y}}_t^{ni}}_{NJ\times 1} = \underbrace{\boldsymbol{\Omega}^{C}}_{NJ\times NNJ}\underbrace{\hat{\boldsymbol{C}}^{CX}}_{NNJ\times 1} + \underbrace{\boldsymbol{\Omega}^{X}}_{NJ\times NJNJ}\underbrace{\hat{\boldsymbol{X}}}_{NJNJ\times 1} \tag{91}$$

Here, for simplification, we will shy away from the sectoral bundles for both production and consumption and we will assume that $\theta = \theta_h = \theta_l^i$ and $\sigma_h = \sigma_l^i$ for all i.[28] From the CES structure we have $\hat{C}_{n,mj,t} = \hat{C}_{n,t} + \sigma_h(\hat{P}_t^C - \hat{P}_{n,mj,t})$. In matrix notation this will be:

$$\underbrace{\hat{\boldsymbol{C}}_t^{nmj}}_{NNJ\times 1} = \underbrace{\boldsymbol{S}_{10}}_{NNJ\times N}\underbrace{\hat{\boldsymbol{C}}_t}_{N\times 1} + \sigma_h\left(\underbrace{\boldsymbol{S}_{11}}_{NNJ\times N}\underbrace{\hat{\boldsymbol{P}}_t^{C}}_{N\times 1} - \underbrace{\hat{\boldsymbol{P}}_t^{CX}}_{NNJ\times 1}\right), \tag{92}$$

where $\hat{\boldsymbol{P}}_t^{CX}$ is the vector of consumption prices. From the production function we have:

$$\underbrace{\hat{\mathbf{Y}}_t^{ni}}_{NJ\times 1} = \underbrace{\boldsymbol{\alpha}}_{NJ\times NJ}\underbrace{\hat{\boldsymbol{L}}_t^{ni}}_{NJ\times 1} + \underbrace{\boldsymbol{\Omega}^{prod}}_{NJ\times NJNJ}\underbrace{\hat{\boldsymbol{X}}}_{NJNJ\times 1} \tag{93}$$

From the CES structure we have $\hat{X}_{ni,mj,t} = \hat{L}_{ni,t} + \theta(\hat{W}_t - \hat{P}_{ni,mj,t})$. In matrix notation

[28] More general case follows the same logic depicted here, but the notation becomes heavily involved.

this will be:

$$\underbrace{\hat{\boldsymbol{X}}_t}_{NJNJ\times 1} = \underbrace{\boldsymbol{S}_{12}}_{NJNJ\times NJ}\underbrace{\hat{\boldsymbol{L}}^{ni}}_{NJ\times 1} + \theta\left(\underbrace{\boldsymbol{S}_{13}}_{NJNJ\times N}\underbrace{\hat{\boldsymbol{W}}_t}_{N\times 1} - \underbrace{\hat{\boldsymbol{P}}_t^X}_{NJNJ\times 1}\right), \tag{94}$$

where $\hat{\boldsymbol{P}}_t^X$ is the vector of input prices. Using (94), we can substitute out $\hat{\boldsymbol{X}}_t$ in (93). Then we shall solve for $\hat{\boldsymbol{L}}_t^{ni}$ in that equation. Call this equation, (95), "labor-output mapping,"

$$\hat{\mathbf{Y}}_t^{ni} = \boldsymbol{\alpha}\,\hat{\boldsymbol{L}}_t^{ni} + \boldsymbol{\Omega}^{prod}\Big(\boldsymbol{S}_{12}\,\hat{\boldsymbol{L}}_t^{ni} + \theta\big[\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\big]\Big)$$
$$\hat{\mathbf{Y}}_t^{ni} = \Big(\boldsymbol{\alpha} + \boldsymbol{\Omega}^{prod}\,\boldsymbol{S}_{12}\Big)\,\hat{\boldsymbol{L}}_t^{ni} + \theta\,\boldsymbol{\Omega}^{prod}\Big(\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\Big)$$

Rearranging and solving for

$$\Big(\boldsymbol{\alpha} + \boldsymbol{\Omega}^{prod}\,\boldsymbol{S}_{12}\Big)\,\hat{\boldsymbol{L}}_t^{ni} = \hat{\mathbf{Y}}_t^{ni} - \theta\,\boldsymbol{\Omega}^{prod}\Big(\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\Big),$$
$$\hat{\boldsymbol{L}}_t^{ni} = \Big(\boldsymbol{\alpha} + \boldsymbol{\Omega}^{prod}\,\boldsymbol{S}_{12}\Big)^{-1}\Big[\hat{\mathbf{Y}}_t^{ni} - \theta\,\boldsymbol{\Omega}^{prod}\Big(\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\Big)\Big] \tag{95}$$

Then we use (94) to substitute out $\hat{\boldsymbol{X}}_t$ and use (92) to substitute out $\hat{\boldsymbol{C}}_t^{nmj}$ in (91). This equation will now have $\hat{\boldsymbol{L}}_t^{ni}$ in it. Substituting that out using "labor-output mapping," we can then solve for $\hat{\mathbf{Y}}_t^{ni}$ and call this "output-consumption mapping." First, substitute (92) and (94) into the market-clearing condition (91). This yields an expression in terms of $\hat{\mathbf{Y}}_t^{ni}$ and $\hat{\mathbf{L}}_t^{ni}$:

$$\hat{\mathbf{Y}}_t^{ni} = \boldsymbol{\Omega}^C\Big[\mathbf{S}_{10}\,\hat{\mathbf{C}}_t + \sigma_h\big(\mathbf{S}_{11}\,\hat{\mathbf{P}}_t^C - \hat{\mathbf{P}}_t^{CX}\big)\Big] + \boldsymbol{\Omega}^X\Big[\mathbf{S}_{12}\,\hat{\mathbf{L}}_t^{ni} + \theta\big(\mathbf{S}_{13}\,\hat{\mathbf{W}}_t - \hat{\mathbf{P}}_t^X\big)\Big].$$

Next, use the labor–output mapping (equation (95)) to substitute out $\hat{\mathbf{L}}_t^{ni}$. Let

$$\mathbf{A} \equiv \boldsymbol{\alpha} + \boldsymbol{\Omega}^{prod}\,\mathbf{S}_{12}.$$

Then

$$\hat{\mathbf{L}}_t^{ni} = \mathbf{A}^{-1}\Big[\hat{\mathbf{Y}}_t^{ni} - \theta\,\boldsymbol{\Omega}^{prod}\big(\mathbf{S}_{13}\,\hat{\mathbf{W}}_t - \hat{\mathbf{P}}_t^X\big)\Big].$$

Substituting this into the above expression and collecting terms in $\hat{\mathbf{Y}}_t^{ni}$ gives

$$\hat{\mathbf{Y}}_t^{ni} = \boldsymbol{\Omega}^C\Big[\mathbf{S}_{10}\,\hat{\mathbf{C}}_t + \sigma_h\big(\mathbf{S}_{11}\,\hat{\mathbf{P}}_t^C - \hat{\mathbf{P}}_t^{CX}\big)\Big]$$
$$+ \boldsymbol{\Omega}^X\Big[\mathbf{S}_{12}\,\mathbf{A}^{-1}\big(\hat{\mathbf{Y}}_t^{ni} - \theta\,\boldsymbol{\Omega}^{prod}(\mathbf{S}_{13}\,\hat{\mathbf{W}}_t - \hat{\mathbf{P}}_t^X)\big) + \theta\big(\mathbf{S}_{13}\,\hat{\mathbf{W}}_t - \hat{\mathbf{P}}_t^X\big)\Big].$$

Rearranging to isolate $\hat{\mathbf{Y}}_t^{ni}$ on the left-hand side and then inverting the resulting coefficient matrix gives us equation, (96), which is the *output–consumption mapping*:

$$\hat{\mathbf{Y}}_t^{ni} = \Big[\mathbf{I} \;-\; \mathbf{\Omega}^X\,\mathbf{S}_{12}\,\mathbf{A}^{-1}\Big]^{-1}\Big\{\mathbf{\Omega}^C\Big[\mathbf{S}_{10}\,\hat{\mathbf{C}}_t + \sigma_h\big(\mathbf{S}_{11}\,\hat{\mathbf{P}}_t^C - \hat{\mathbf{P}}_t^{CX}\big)\Big] \;+\; \theta\,\mathbf{\Omega}^X\Big[\mathbf{S}_{13}\,\hat{\mathbf{W}}_t - \hat{\mathbf{P}}_t^X\Big] \\ -\;\theta\,\mathbf{\Omega}^X\,\mathbf{S}_{12}\,\mathbf{A}^{-1}\,\mathbf{\Omega}^{prod}\Big[\mathbf{S}_{13}\,\hat{\mathbf{W}}_t - \hat{\mathbf{P}}_t^X\Big]\Big\}. \tag{96}$$

We now return to (88). Let us substitute out $\hat{\mathbf{X}}_t$ in that equation using (94). Next we substitute $\hat{\mathbf{L}}_t^{ni}$ in the resulting expression using (95). Finally we substitute out $\hat{\mathbf{Y}}_t^{ni}$ in the resulting expression using (96), ending up with an expression that expresses net exports as a function of the aggregate consumption vector and prices. Recalling that we defined

$$\mathbf{A} \;\equiv\; \boldsymbol{\alpha} \;+\; \mathbf{\Omega}^{prod}\,\boldsymbol{S}_{12}, \quad \text{and} \quad \hat{\boldsymbol{L}}_t^{ni} = \mathbf{A}^{-1}\Big[\hat{\boldsymbol{Y}}_t^{ni} \;-\; \theta\,\mathbf{\Omega}^{prod}\big(\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t \;-\; \hat{\boldsymbol{P}}_t^X\big)\Big].$$

From the *output–consumption mapping* (96), we have

$$\hat{\boldsymbol{Y}}_t^{ni} = \Big[\mathbf{I} \;-\; \mathbf{\Omega}^X\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\Big]^{-1}\Big\{\mathbf{\Omega}^C\Big[\boldsymbol{S}_{10}\,\hat{\boldsymbol{C}}_t \;+\; \sigma_h\big(\boldsymbol{S}_{11}\,\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{CX}\big)\Big] \\ +\,\theta\,\mathbf{\Omega}^X\Big[\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\Big] - \theta\,\mathbf{\Omega}^X\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\mathbf{\Omega}^{prod}\Big[\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\Big]\Big\}.$$

Starting again from (88),

$$\widehat{NX}_{n,t} = \boldsymbol{S}_1\big(\hat{\boldsymbol{Y}}_t^{ni} + \hat{\boldsymbol{P}}_t^P\big) \;-\; \boldsymbol{S}_2\big(\hat{\boldsymbol{C}}_t + \hat{\boldsymbol{P}}_t^C\big) \;-\; \boldsymbol{S}_3\big(\hat{\boldsymbol{X}}_t + \hat{\boldsymbol{P}}_t^X\big),$$

we substitute (94) for $\hat{\boldsymbol{X}}_t$, then (95) for $\hat{\boldsymbol{L}}_t^{ni}$, and finally (96) for $\hat{\boldsymbol{Y}}_t^{ni}$. Inserting each expression carefully and gathering terms gives:

$$\widehat{NX}_{n,t} = \boldsymbol{S}_1\Bigg(\Big[\mathbf{I} - \mathbf{\Omega}^X\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\Big]^{-1}\Big\{\mathbf{\Omega}^C\Big[\boldsymbol{S}_{10}\,\hat{\boldsymbol{C}}_t + \sigma_h\big(\boldsymbol{S}_{11}\,\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{CX}\big)\Big] \\ +\,\theta\,\mathbf{\Omega}^X\Big[\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\Big] - \theta\,\mathbf{\Omega}^X\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\mathbf{\Omega}^{prod}\Big[\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^X\Big]\Big\} \;+\; \hat{\boldsymbol{P}}_t^P\Bigg) \\ -\,\boldsymbol{S}_2\Big(\hat{\boldsymbol{C}}_t + \hat{\boldsymbol{P}}_t^C\Big) \\ -\,\boldsymbol{S}_3\Bigg[\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\bigg(\Big[\mathbf{I} - \mathbf{\Omega}^X\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\Big]^{-1}\Big\{\mathbf{\Omega}^C\Big[\boldsymbol{S}_{10}\,\hat{\boldsymbol{C}}_t + \sigma_h\big(\boldsymbol{S}_{11}\,\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{nmj}\big)\Big] \\ +\,\theta\,\mathbf{\Omega}^X\Big[\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^{nimj}\Big] - \theta\,\mathbf{\Omega}^X\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\mathbf{\Omega}^{prod}\Big[\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^{nimj}\Big]\Big\}$$

$$- \; \theta\, \boldsymbol{\Omega}^{prod}\Big(\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^{nimj}\Big)\Big) \;+\; \theta\Big(\boldsymbol{S}_{13}\,\hat{\boldsymbol{W}}_t - \hat{\boldsymbol{P}}_t^{nimj}\Big) \;+\; \hat{\boldsymbol{P}}_t^{nimj}\Bigg].$$

This final expression shows $\widehat{NX}_{n,t}$ as a function of the aggregate consumption vector, the wage vector, and the relevant price vectors. In our analytical solution we use $\hat{\boldsymbol{W}}_t = \hat{\boldsymbol{P}}_t^C + \hat{\boldsymbol{C}}_t$, so we plug that in. Defining $\widetilde{A} = \big[\mathbf{I} \;-\; \boldsymbol{\Omega}^X\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\big]^{-1}$ we can multiply terms out and rearrange:

$$\begin{aligned}
\widehat{NX}_{n,t} = \Big(&\boldsymbol{S}_1\,\widetilde{A}\,\boldsymbol{\Omega}^C\,\boldsymbol{S}_{10} \;+\; \boldsymbol{S}_1\,\widetilde{A}\,\theta\,\boldsymbol{\Omega}^X\,\boldsymbol{S}_{13} \;-\; \boldsymbol{S}_1\,\widetilde{A}\,\theta\,\boldsymbol{\Omega}^X\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\boldsymbol{\Omega}^{prod}\,\boldsymbol{S}_{13} \\
&- \boldsymbol{S}_2 \;-\; \boldsymbol{S}_3\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\widetilde{A}\,\boldsymbol{\Omega}^C\,\boldsymbol{S}_{10} \;-\; \boldsymbol{S}_3\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\widetilde{A}\,\theta\,\boldsymbol{\Omega}^X\,\boldsymbol{S}_{13} \\
&+ \boldsymbol{S}_3\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\widetilde{A}\,\theta\,\boldsymbol{\Omega}^X\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\boldsymbol{\Omega}^{prod}\,\boldsymbol{S}_{13} \;+\; \boldsymbol{S}_3\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\theta\,\boldsymbol{\Omega}^{prod}\,\boldsymbol{S}_{13} \;-\; \theta\,\boldsymbol{S}_3\,\boldsymbol{S}_{13}\Big)\,\hat{\boldsymbol{C}}_t \\
+ \;\Big(&\boldsymbol{S}_1\,\widetilde{A}\,\boldsymbol{\Omega}^C\,\sigma_h\,\boldsymbol{S}_{11} \;+\; \boldsymbol{S}_1\,\widetilde{A}\,\theta\,\boldsymbol{\Omega}^X\,\boldsymbol{S}_{13} \;-\; \boldsymbol{S}_1\,\widetilde{A}\,\theta\,\boldsymbol{\Omega}^X\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\boldsymbol{\Omega}^{prod}\,\boldsymbol{S}_{13} \\
&- \boldsymbol{S}_2 \;-\; \boldsymbol{S}_3\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\widetilde{A}\,\boldsymbol{\Omega}^C\,\sigma_h\,\boldsymbol{S}_{11} \;-\; \boldsymbol{S}_3\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\widetilde{A}\,\theta\,\boldsymbol{\Omega}^X\,\boldsymbol{S}_{13} \\
&+ \boldsymbol{S}_3\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\widetilde{A}\,\theta\,\boldsymbol{\Omega}^X\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\boldsymbol{\Omega}^{prod}\,\boldsymbol{S}_{13} \;-\; \boldsymbol{S}_3\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\theta\,\boldsymbol{\Omega}^{prod}\,\boldsymbol{S}_{13} \;-\; \theta\,\boldsymbol{S}_3\,\boldsymbol{S}_{13}\Big)\,\hat{\boldsymbol{P}}_t^C \\
+ \;\Big(&\boldsymbol{S}_1\Big)\,\hat{\boldsymbol{P}}_t^P \\
+ \;\Big(&-\boldsymbol{S}_1\,\widetilde{A}\,\boldsymbol{\Omega}^C\,\sigma_h \;+\; \boldsymbol{S}_3\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\widetilde{A}\,\boldsymbol{\Omega}^C\,\sigma_h \;+\; \sigma_h\,\boldsymbol{S}_3\Big)\,\hat{\boldsymbol{P}}_t^{CX} \\
+ \;\Big(&-\boldsymbol{S}_1\,\widetilde{A}\,\theta\,\boldsymbol{\Omega}^X \;+\; \boldsymbol{S}_1\,\widetilde{A}\,\theta\,\boldsymbol{\Omega}^X\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\boldsymbol{\Omega}^{prod} \\
&+ \boldsymbol{S}_3\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\widetilde{A}\,\theta\,\boldsymbol{\Omega}^X \;-\; \boldsymbol{S}_3\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\widetilde{A}\,\theta\,\boldsymbol{\Omega}^X\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\boldsymbol{\Omega}^{prod} \\
&- \boldsymbol{S}_3\,\boldsymbol{S}_{12}\,\mathbf{A}^{-1}\,\theta\,\boldsymbol{\Omega}^{prod} \;+\; (\theta \;-\; 1)\,\boldsymbol{S}_3\Big)\,\hat{\boldsymbol{P}}_t^X.
\end{aligned}$$

Given the definitions for $\hat{\boldsymbol{P}}_t^X$ and $\hat{\boldsymbol{P}}_t^{CX}$, as these are linear combinations of producer prices, exchange rate and tariffs, and given that the US nominal interest rate is a function of US price level, we can thus write:

$$\begin{aligned}
\beta\hat{V}_{n,t}^{US} - \hat{V}_{n,t-1}^{US} &= (1-\beta)\hat{\mathcal{E}}_{n,t} - (1-\beta)\widehat{NX}_{n,t} + \beta\hat{i}_t^{US} \\
\beta\hat{V}_{n,t}^{US} &= \Gamma_1\hat{V}_{n,t-1}^{US} + \boldsymbol{\Gamma}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Gamma}_3\hat{\boldsymbol{P}}_t^P + \Gamma_4\mathcal{E}_t + \Gamma_5\tau_t
\end{aligned}$$

where $\Gamma_1 = 1$ in the case of the two-country model; aggregating this yields the fifth equation in the five-equation representation. From the expression above and from intuition, we can see that a higher elasticity of substitution makes the balance of payments more reactive to changes in prices. More broadly we see net exports react to the aggregate demand stance of countries and the terms of trade.

Stacking the final expression above for different countries n, alongside a market-clearing condition for US bonds, yields the fifth equation in the five-equation Global New Keynesian Representation.

F Decomposing the Impact on Inflation

Starting with Equation (44) we can write:

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \boldsymbol{\Xi}\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda}\left[\boldsymbol{L}_\tau^P + \left(\boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi}) + \beta(\boldsymbol{L}_C^P + \boldsymbol{L}_\mathcal{E}^P)\boldsymbol{\Phi}\boldsymbol{L}_\mathcal{E}^C\right)\boldsymbol{L}_\tau^C\right] + \boldsymbol{L}_\tau^C$$

Rearranging:

$$\begin{aligned}
\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} =& \boldsymbol{\Xi}\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda}\boldsymbol{L}_\tau^P + \boldsymbol{\Xi}\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda}\boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{L}_\tau^C \\
&\quad + \beta\boldsymbol{\Xi}\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda}\boldsymbol{L}_C^P\boldsymbol{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C + \beta\boldsymbol{\Xi}\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda}\boldsymbol{L}_\mathcal{E}^P\boldsymbol{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C + \boldsymbol{L}_\tau^C \\
\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} =& \boldsymbol{\Xi}\Big(\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda}\boldsymbol{L}_\tau^P + \tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda}\boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{L}_\tau^C \\
&\quad + \beta\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda}\boldsymbol{L}_C^P\boldsymbol{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C + \beta\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda}\boldsymbol{L}_\mathcal{E}^P\boldsymbol{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C\Big) + \boldsymbol{L}_\tau^C \\
\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} =& \boldsymbol{\Xi}\left(\underbrace{\boldsymbol{L}_\tau^P + \boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{L}_\tau^C + \beta\boldsymbol{L}_C^P\boldsymbol{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C + \beta\boldsymbol{L}_\mathcal{E}^P\boldsymbol{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C}_{\mathbf{Z}} + (\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda} - I)\mathbf{Z}\right) \\
&+ \boldsymbol{L}_\tau^C \\
\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} =& \boldsymbol{\Xi}\boldsymbol{L}_\tau^P + \boldsymbol{\Xi}\boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{L}_\tau^C + \beta\boldsymbol{\Xi}\boldsymbol{L}_C^P\boldsymbol{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C + \beta\boldsymbol{\Xi}\boldsymbol{L}_\mathcal{E}^P\boldsymbol{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C + \boldsymbol{L}_\tau^C \\
&+ \boldsymbol{\Xi}(\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda} - I)\mathbf{Z}
\end{aligned}$$

This is the desired decomposition:

$$\begin{aligned}
\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} =& \underbrace{\boldsymbol{\Xi}\boldsymbol{L}_\tau^P}_{\text{Direct PPI effect}} + \underbrace{\boldsymbol{\Xi}\boldsymbol{L}_C^P(\mathbf{I}-\boldsymbol{\Phi})\boldsymbol{L}_\tau^C}_{\text{Demand channel}} + \underbrace{\beta\boldsymbol{\Xi}\boldsymbol{L}_C^P\boldsymbol{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C}_{\text{Expected demand channel}} + \underbrace{\beta\boldsymbol{\Xi}\boldsymbol{L}_\mathcal{E}^P\boldsymbol{\Phi}\boldsymbol{L}_\mathcal{E}^C\boldsymbol{L}_\tau^C}_{\text{Expected ER channel}} + \underbrace{\boldsymbol{L}_\tau^C}_{\text{Direct CPI effect}} \\
&+ \underbrace{\boldsymbol{\Xi}(\tilde{\boldsymbol{\Psi}}_\phi^{NKOE}\boldsymbol{\Lambda} - \mathbf{I})\mathbf{Z}}_{\text{Propagation}}
\end{aligned}$$

G Impact Under Higher Elasticity of Substitution

Below, we present the impulse response functions for a scenario in which the Armington elasticity of substitution is set to 4, following the assumption used by USTR (2025). Compared to the results shown in Figure 7, the responses of real variables—such as real GDP, net exports as a share of GDP, and unemployment—are more muted. Likewise, the immediate effects on nominal exchange rates and inflation are also reduced in magnitude. This case is assumed to be the more optimistic case for the proponents of tariffs and even with more muted responses from variables, the US nevertheless suffers from higher inflation and a drop in Real GDP, while the improvement in net exports as a share of GDP is limited to approximately 0.5% of GDP.

Figure 11. Impact of All-Out Tariff War under EoS> 1